OPERATII GEOMETRICE

C. VERTAN

O definitie ?

Clasa de operatii ce modifica structura de vecinatate a pixelilor din imagine (structura organizarii spatiale a imaginii).

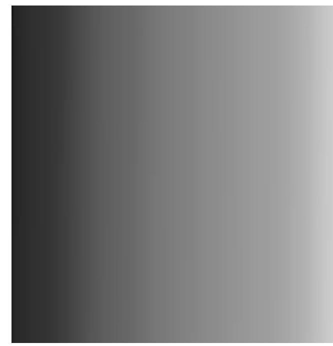
Transformarile geometrice permit deplasarea pixelilor in imagine, pe noi pozitii.

Model intuitiv: imaginea este imprimata pe o foaie subtire de cauciuc, care poate fi deformata oricum.

Operatii geometrice fara constrangeri

Daca miscarea oricarui pixel este impusa in mod independent de miscarea celorlalti pixeli, poate apare un efect de amestecare (sau de rupere) a continutului vizual.

sferturile imaginii sunt interschimbate



mutare “aleatore” a pixelilor

Constrangere: lege de descriere a miscarii pixelilor NU este aleatoare si este invarianta spatial (toti pixelii au o miscare descrisa de aceleasi ecuatii).

Definirea transformarii geometrice

Transformarea = ecuatia de modificare a coordonatelor pixelilor

$$x' = X(x, y)$$
$$y' = Y(x, y)$$

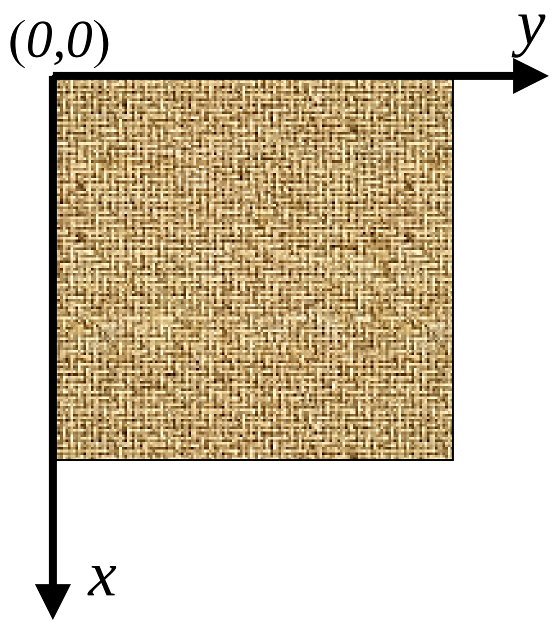
x’ , y’ :
coordonatele pixelilor
dupa transfomare

X, Y:
ecuatiile de modificare
a coordonatelor

x, y :
coordonatele pixelilor
inainte de transfomare

Formele particulare ale functiilor de transformare a coordonatelor, *X(x,y)* si *Y(x,y)* particularizeaza transformarile geometrice.

Sistemul de coordonate asociat imaginii

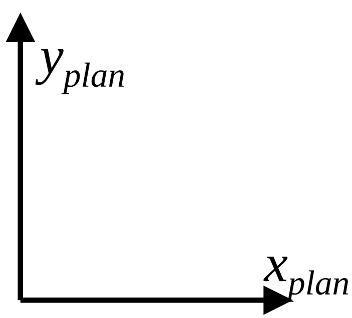


Originea este coltul din stanga sus, coordonatele cresc de la stanga la dreapta si de sus in jos.

Prima coordonata este de linie.

Coordonatele pot fi continue (x, y) sau discrete (m, n)

Transformarea la sistemul cartezian plan uzual :



$$x_{plan} = y$$

$$y_{plan} = -x$$

Transformari geometrice afine elementare

Translatia

Deplasarea in plan a continutului imaginii;
echivalenta cu schimbarea originii sistemului
de coordonate atasat imaginii.

$$x' = X(x, y) = x + x_0$$

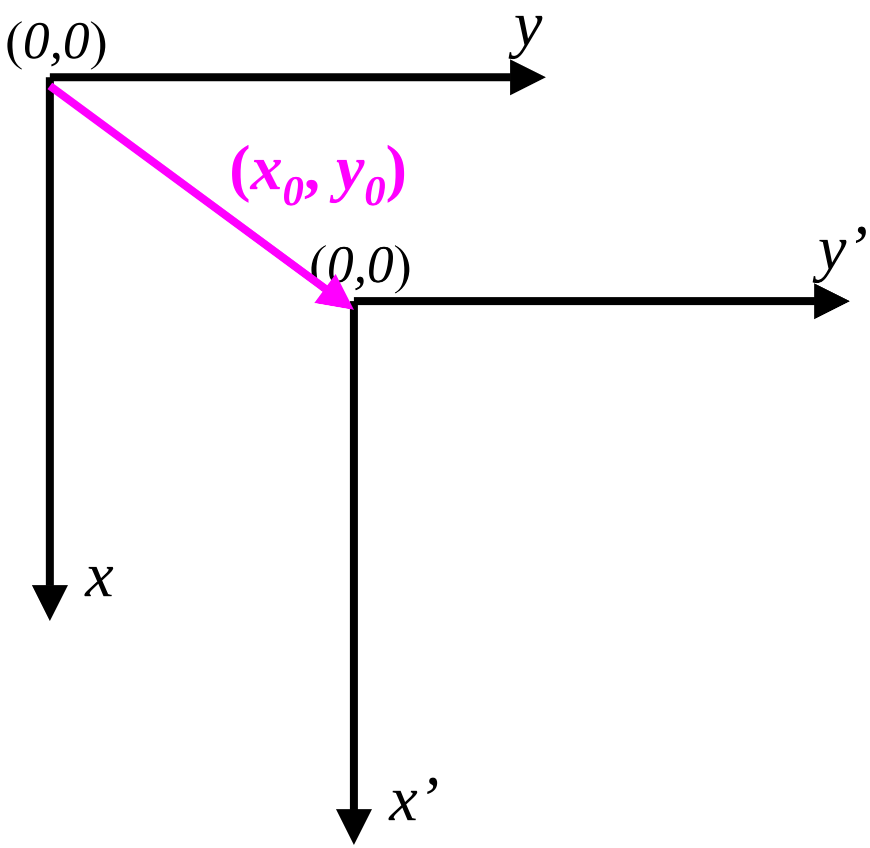
$$y' = Y(x, y) = y + y_0$$

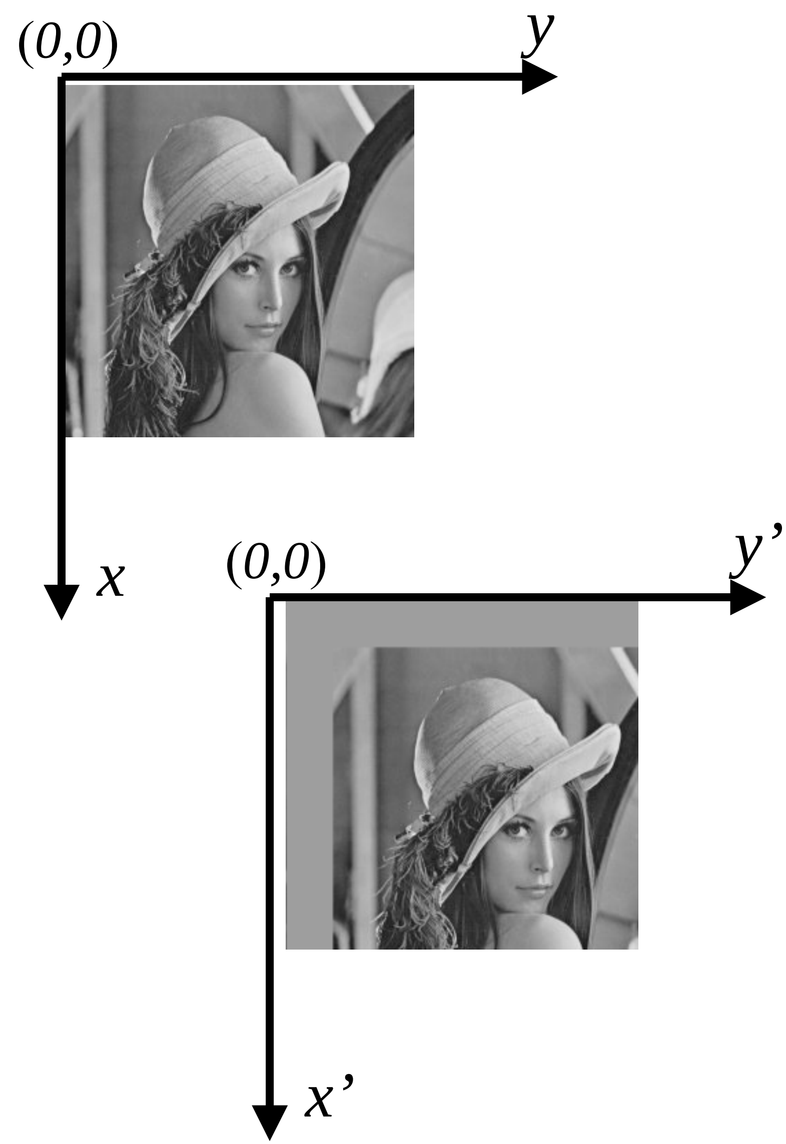
Parametri : x_0, y_0 – deplasari pe verticala/ orizontala.

Pastreaza distantele dintre pixeli.

Transformari geometrice afine elementare

Translatia





Transformari geometrice afine elementare

Scalarea

Intinderea/ comprimarea continutului imaginii dupa una sau ambele axe de coordonate.
NU pastreaza distantele dintre pixeli

$$x' = X(x, y) = \alpha x$$

$$y' = Y(x, y) = \beta y$$

$$\alpha, \beta > 0$$

Parametri : α, β - factori de scalare pe verticala/ orizontala

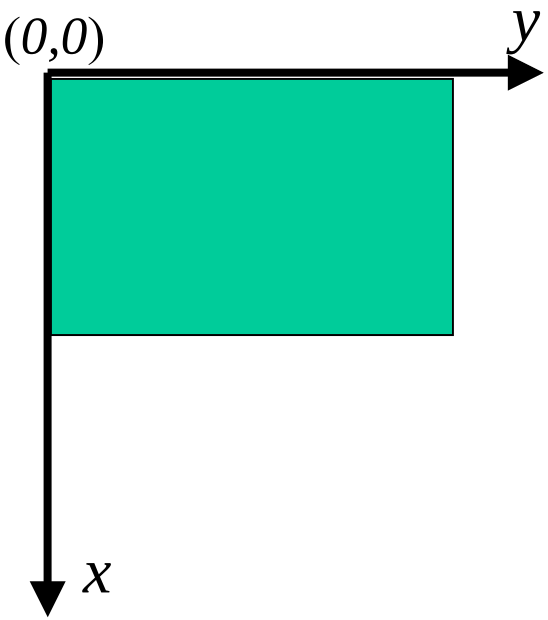
$\alpha, \beta > 1$: marire/ intindere

$\alpha, \beta < 1$: micsorare/ comprimare

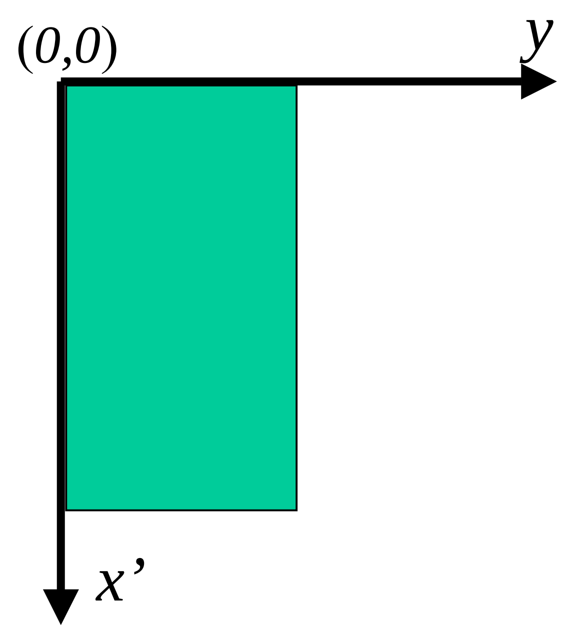
$\alpha = \beta$: scalare omogena

Transformari geometrice afine elementare

Scalarea



$\alpha > 1,\ \beta < 1$



$\alpha < 1,\ \beta > 1$

Transformari geometrice afine elementare

Inclinarea

Inclinarea imaginii reprezinta deplasarea pixelilor dupa o singura coordonata, dependent de pozitia globala in imagine, cealalta coordonata ramanand nemodificata .

$$x' = X(x, y) = x$$
$$y' = Y(x, y) = tx + y$$
$$t > 0$$

inclinare pe orizontala

$$x' = X(x, y) = x + sy$$
$$y' = Y(x, y) = y$$
$$s > 0$$

inclinare pe verticala

Parametri : t, s – coeficienti de inclinare.

NU pastreaza distantele dintre pixeli.

Transformari geometrice afine elementare

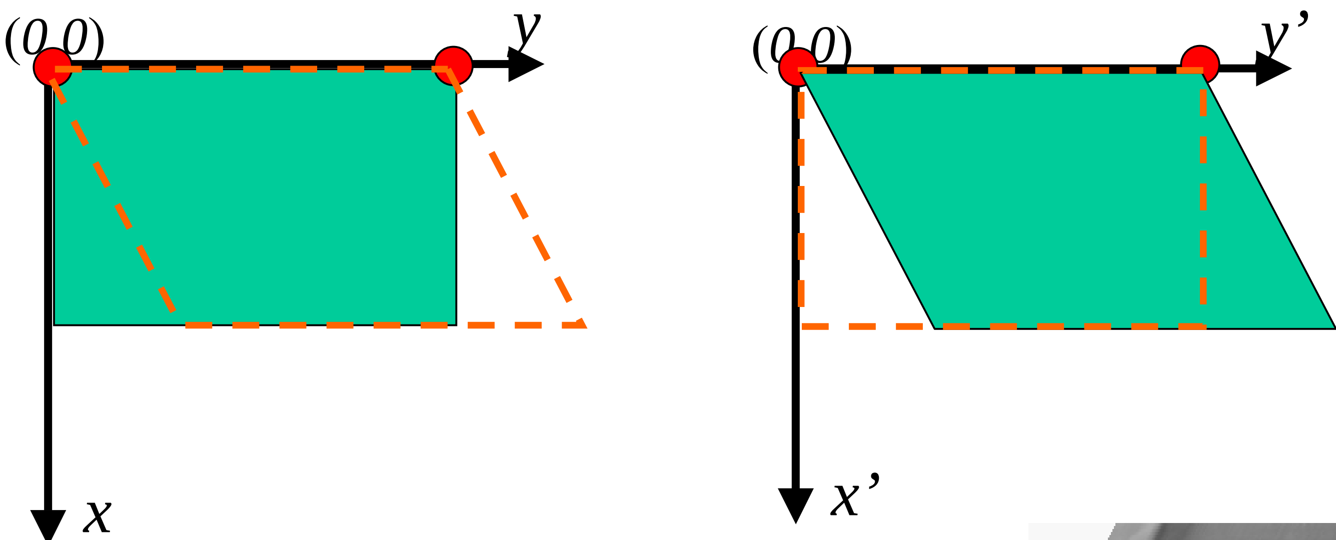
Inclinarea pe orizontala

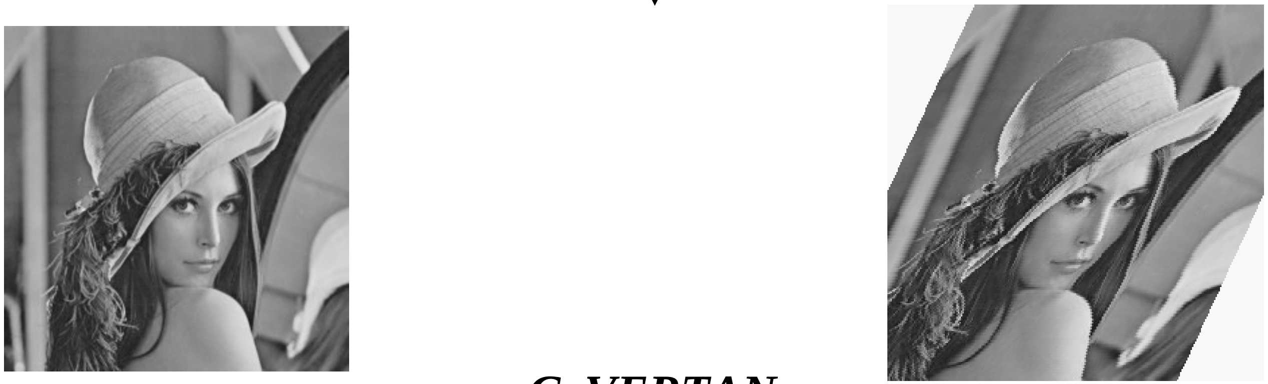
$$x' = X(x, y) = x$$

$$y' = Y(x, y) = tx + y$$

$$t > 0$$

punctele de *x=0* sunt puncte fixe



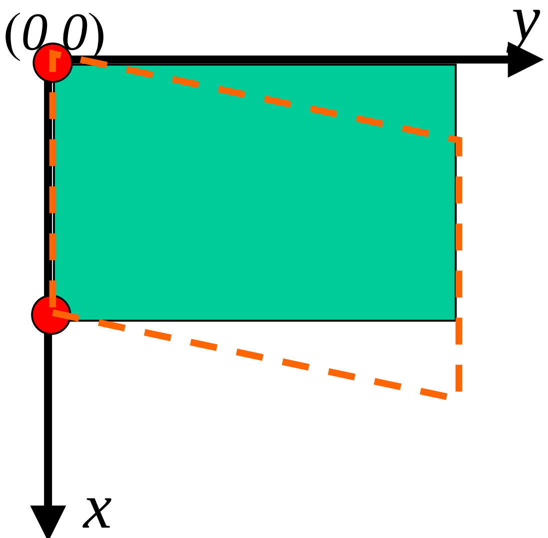


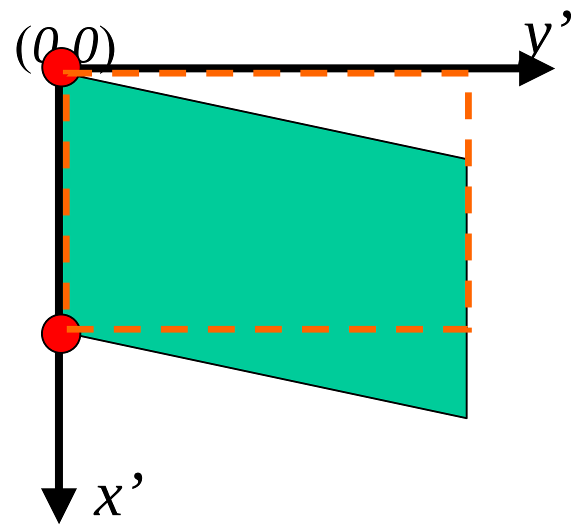
Transformari geometrice afine elementare

$$x' = X(x, y) = x + sy$$
$$y' = Y(x, y) = y$$
$$s > 0$$

Inclinarea pe verticala

- punctele de *y=0* sunt puncte fixe





Transformari geometrice afine elementare

Rotatia

Deplasare circulara a pixelilor in jurul centrului de rotatie (originea sistemului de coordonate).

$$x' = X(x,y) = x\cos\theta + y\sin\theta$$

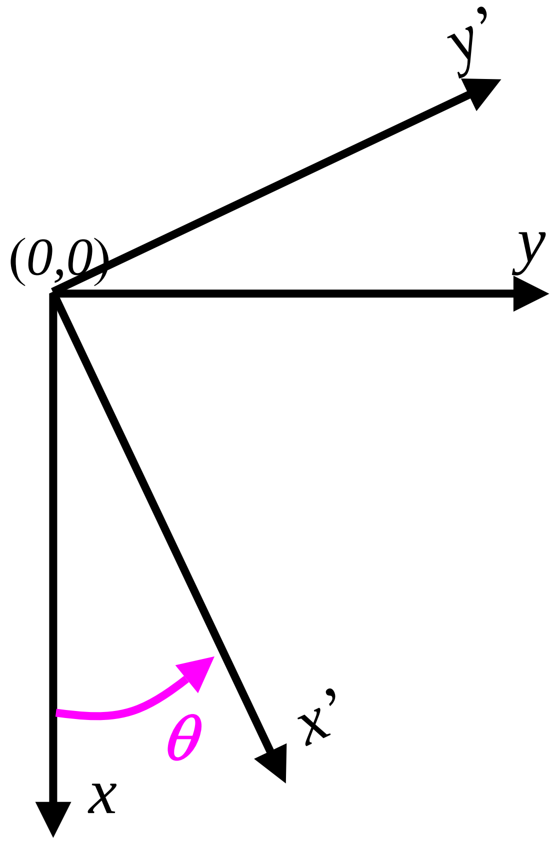
$$y' = Y(x,y) = -x\sin\theta + y\cos\theta$$

Parametru : θ - unghiul de rotatie

Pastreaza distantele dintre pixeli.

Transformari geometrice afine elementare

Rotatia



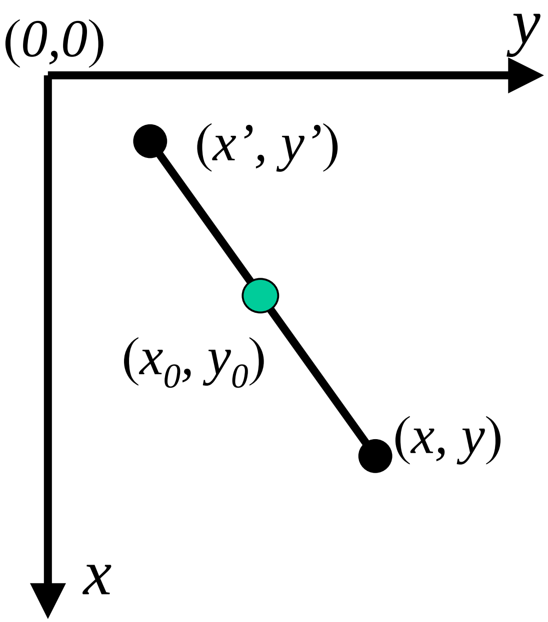
Transformari geometrice afine compuse

Reflexia fata de un centru de reflexie (x_0,y_0)

Pozitie initiala si pozitia finala a fiecarul pixel formeaza un segment de dreapta al carui centru este centrul de reflexie.

$$x' = X(x, y) = 2x_0 - x$$

$$y' = Y(x, y) = 2y_0 - y$$



Transformari geometrice afine compuse

Reflexia fata de o axa de reflexie

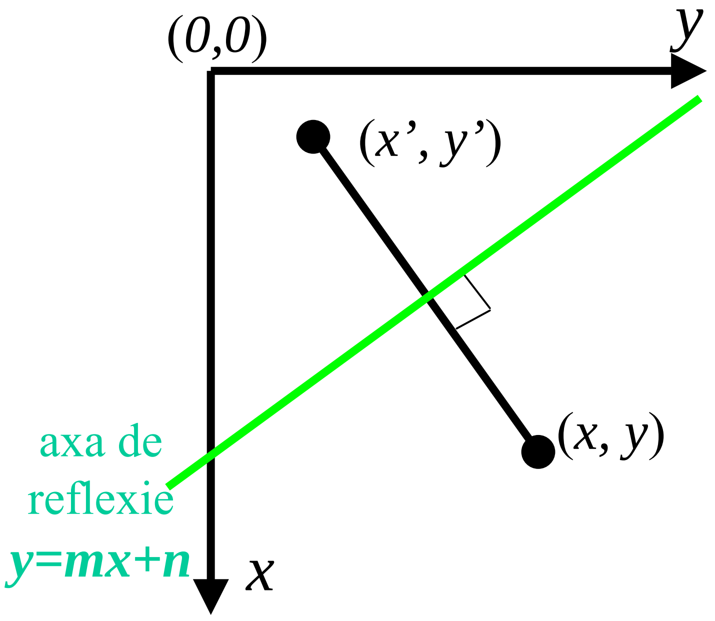
Pozitie initiala si pozitia finala a fiecarul pixel formeaza un segment de dreapta a carui mediatoare este axa de reflexie.

$$\frac{y'-y}{x'-x} = -\frac{1}{m}$$

$$\frac{|y-mx-n|}{\sqrt{m^2+1}} = \frac{|y'-mx'-n|}{\sqrt{m^2+1}}$$

$$x' = X(x,y) = \frac{1-m^2}{1+m^2}x + \frac{2m}{1+m^2}y - \frac{2mn}{1+m^2}$$

$$y' = Y(x,y) = \frac{2m}{1+m^2}x - \frac{1-m^2}{1+m^2}y + \frac{2n}{1+m^2}$$



Transformari geometrice afine compuse

Reflexia fata de o axa de reflexie : cazuri particulare

axa orizontala: $x = k = ct$ ($1/m = 0$, $-n/m = k$)

$$x' = X(x,y) = \frac{1-m^2}{1+m^2}x + \frac{2m}{1+m^2}y - \frac{2mn}{1+m^2}$$

$$y' = Y(x,y) = \frac{2m}{1+m^2}x - \frac{1-m^2}{1+m^2}y + \frac{2n}{1+m^2}$$

$$x' = X(x,y) = 2k - x$$

$$y' = Y(x,y) = y$$

axa verticala: $y = k = ct$ ($m=0$, $n=k$)

$$x' = X(x,y) = \frac{1-m^2}{1+m^2}x + \frac{2m}{1+m^2}y - \frac{2mn}{1+m^2}$$

$$y' = Y(x,y) = \frac{2m}{1+m^2}x - \frac{1-m^2}{1+m^2}y + \frac{2n}{1+m^2}$$

$$x' = X(x,y) = x$$

$$y' = Y(x,y) = 2k - y$$

Forma matriciala a transformarilor geometrice elementare

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = A\begin{pmatrix} x \\ y \end{pmatrix} + B$$

1. Translatia

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$$

2. Scalarea

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \alpha & 0 \\ 0 & \beta \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$

3a. Inclinarea pe orizontala

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ t & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$

3b. Inclinarea pe verticala

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & s \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$

4. Rotatia

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$

Forma matriciala a transformarilor geometrice elementare

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = A \begin{pmatrix} x \\ y \end{pmatrix} + B$$

transformari compuse prin translatie si rotatie

1. Reflexia fata de un punct

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 2x_0 \\ 2y_0 \end{pmatrix}$$

2. Reflexia fata de o axa

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \frac{1-m^2}{1+m^2} & \frac{2m}{1+m^2} \\ \frac{2m}{1+m^2} & -\frac{1-m^2}{1+m^2} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} -\frac{2mn}{1+m^2} \\ \frac{2m}{1+m^2} \end{pmatrix}$$

Transformari geometrice compuse

Compunere = iterare de transformari elementare

Din punctul de vedere al exprimarii analitice, compunerea este simpla in forma matriciala ...

dar translatia nu are aceeasi forma matriciala si nu poate fi exprimata ca un produs matricial.

Solutie : coordonatele omogene

$$\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

(planul de $z=1$ din spatiul cartezian 3D)

Forma matriciala omogena a transformarilor geometrice

1. Translatia

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & x_0 \\ 0 & 1 & y_0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

2. Scalarea

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} \alpha & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

3a. Inclinarea pe orizontala

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ t & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

3b. Inclinarea pe verticala

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & s & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

4. Rotatia

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = A \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

Descompunerea operatiilor geometrice complexe

Sub forma cea mai generala, in coordonate omogene avem:

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & x_0 \\ a_{21} & a_{22} & y_0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

Coeficientii a_{ij} provin numai din scalare, rotatie si inclinare.

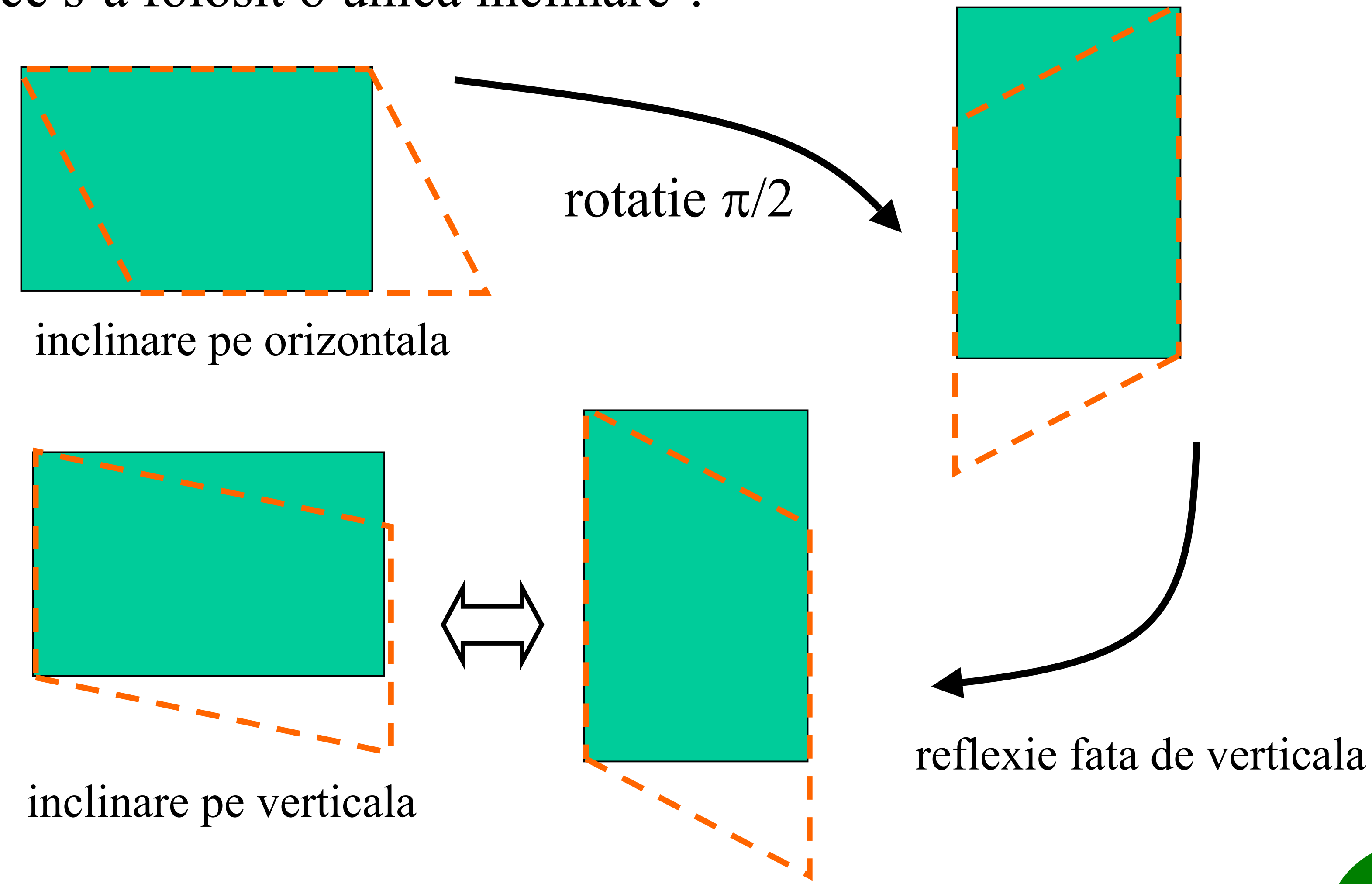
$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \begin{pmatrix} \alpha & 0 \\ 0 & \beta \end{pmatrix} \begin{pmatrix} 1 & s \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

dandu-se o serie de transf. elementare, se gaseste o transf. echivalenta unica.

dandu-se o transf. afina, se gasesc transf. elementare din care e constituita

Descompunerea operatiilor geometrice complexe

De ce s-a folosit o unica inclinare ?



Descompunerea operatiilor geometrice complexe

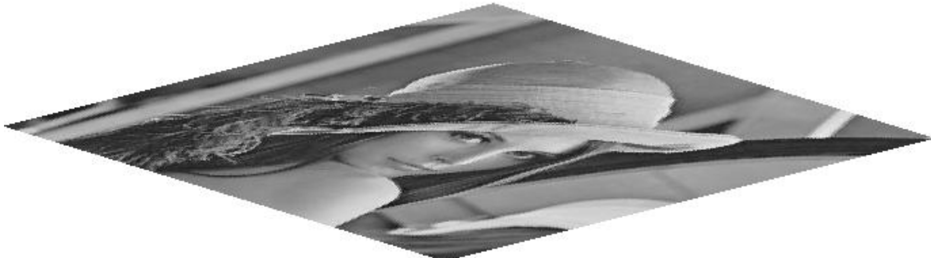
Problema triviala. ← dandu-se o serie de transf. elementare, se gaseste o transf. echivalenta unica.

Ex. Transformarea echivalenta unei rotatii cu 45º (θ=45º), inclinare verticala cu coeficient de 0.1 ($s=0.1$), scalare orizontala de 2 ($\beta=2$) si scalare verticala de 0.5 ($\alpha=0.5$).

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \begin{pmatrix} \alpha & 0 \\ 0 & \beta \end{pmatrix} \begin{pmatrix} 1 & s \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \begin{pmatrix} 0.5 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} 1 & 0.1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \sqrt{2}/2 & \sqrt{2}/2 \\ -\sqrt{2}/2 & \sqrt{2}/2 \end{pmatrix}$$

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \begin{pmatrix} 0.3535 & 0.3889 \\ -1.4142 & 1.2728 \end{pmatrix}$$



Descompunerea operatiilor geometrice complexe

dandu-se o transf. afina,
se gasesc transf. elementare
din care e constituita

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \begin{pmatrix} \alpha\cos\theta - s\sin\theta & \alpha\sin\theta + \beta\cos\theta \\ -\beta\sin\theta & \beta\cos\theta \end{pmatrix}$$

$$\beta = \sqrt{a_{21}^2 + a_{22}^2} \qquad \theta = -\arctan\frac{a_{21}}{a_{22}}$$

$$\alpha = \frac{\sqrt{a_{21}^2 + a_{22}^2}}{a_{21}}(a_{12} - a_{22})$$

$$s = \frac{a_{11}}{a_{21}}\sqrt{a_{21}^2 + a_{22}^2} - a_{22}\frac{\sqrt{a_{21}^2 + a_{22}^2}}{a_{21}}(a_{12} - a_{22})$$

Transformari geometrice neliniare

deplasari controlate dupa
o lege sinusoidala

Efectul de perna

efect de perna pe orizontala

$$x' = X(x,y) = \frac{H}{2} + \left(y - \frac{H}{2}\right)\left(1 - \frac{2}{H} A_x \sin \pi \frac{y}{W}\right)$$

$$y' = Y(x,y) = y$$

efect de perna pe verticala

$$x' = X(x,y) = x$$

$$y' = Y(x,y) = \frac{W}{2} + \left(x - \frac{W}{2}\right)\left(1 - \frac{2}{W} A_y \sin \pi \frac{x}{H}\right)$$

Implementarea operatiilor geometrice

In practica, exista doua componente:

1. unde se “muta” fiecare pixel

transformarea geometrica

2. ce valoare este plasata in noua pozitie ?

interpolarea

In practica, imaginile sunt discrete: coordonatele tuturor pixelilor sunt numere **intregi**.

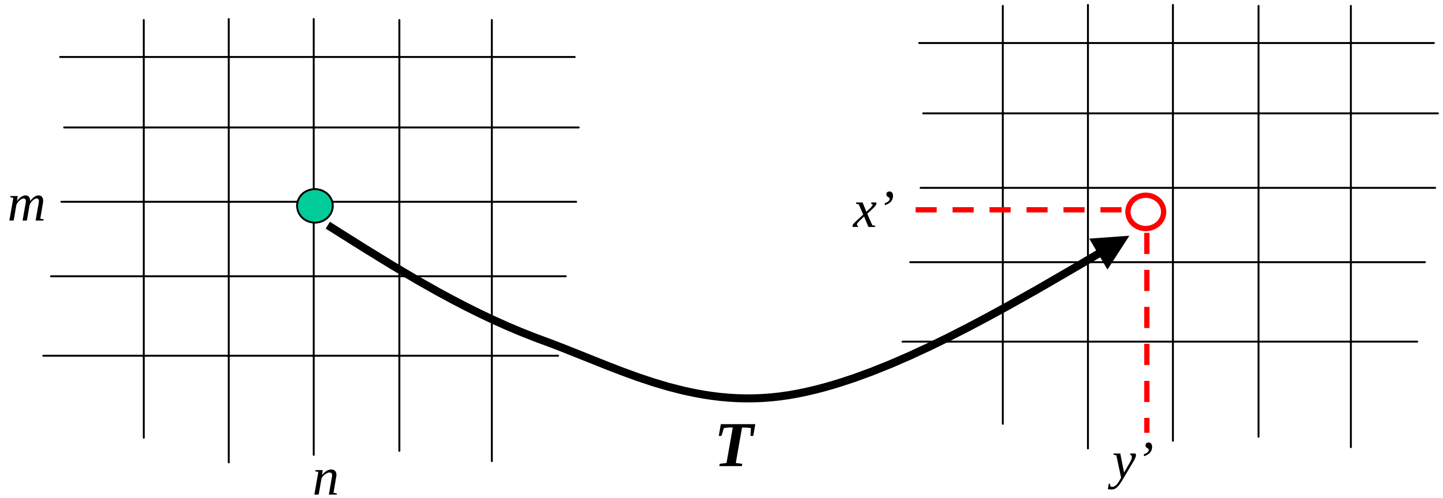
Valorile coordonatelor ce se obtin dupa transformarea geometrica sunt de cele mai multe ori numere **reale**.

Rezulta necesitatea de a introduce o modalitate de a “fabrica” coordonate intregi.

Implementarea operatiilor geometrice

Varianta 1: transportul pixelilor (*pixel carry-over*)

Varianta 2: umplerea pixelilor (*pixel filling*)

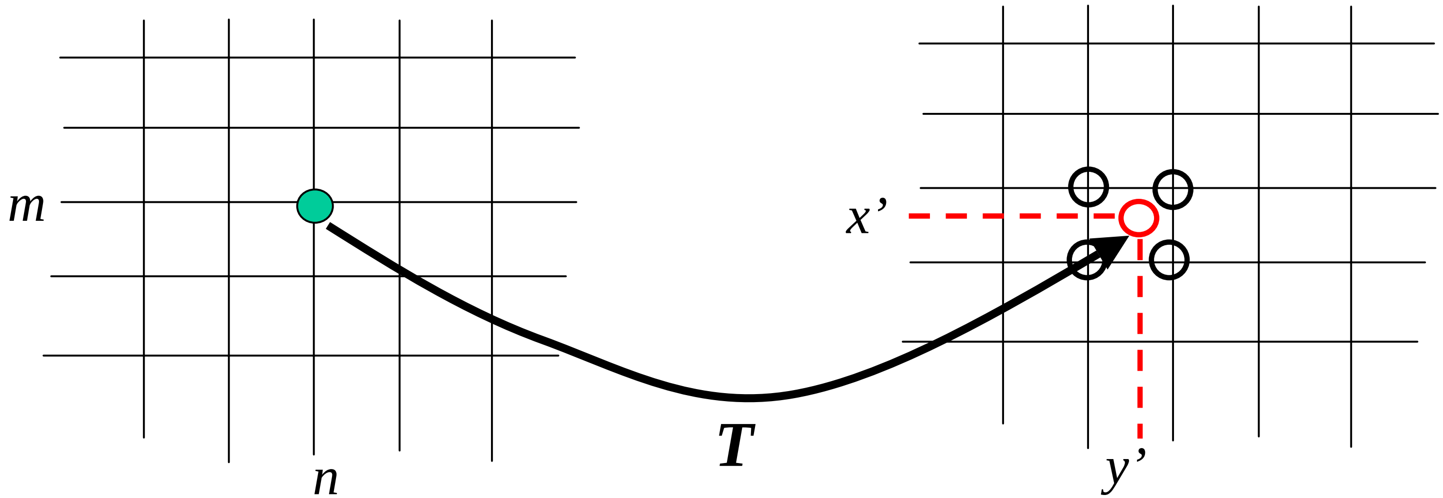


(m, n) coordonate intregi inainte de transformare

(x', y') coordonate ne-intregi dupa transformare

Implementarea operatiilor geometrice

Transportul pixelilor (*pixel carry-over*)



(m, n) coordonate intregi inainte de transformare

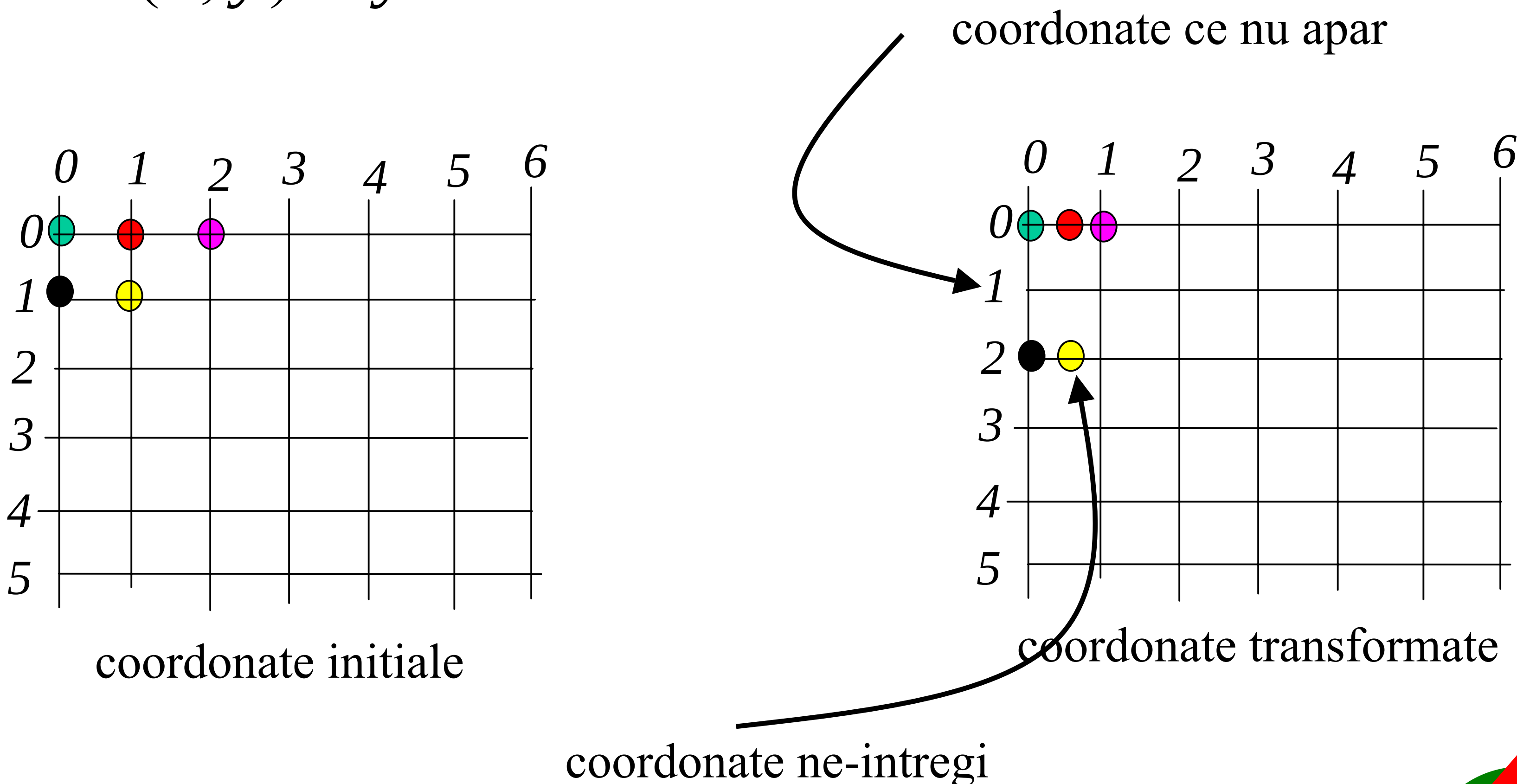
(x', y') coordonate ne-intregi dupa transformare

valoarea trebuie **distribuita** la pixelii vecini

Probleme ale implementarii directe

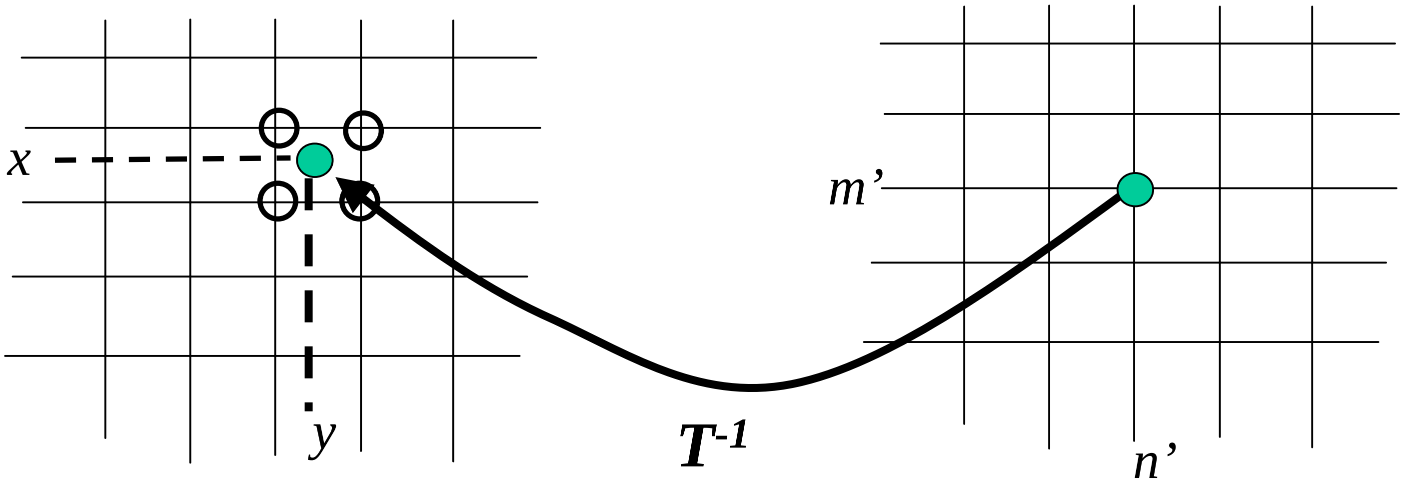
$$x' = X(x, y) = 2x$$

$$y' = Y(x, y) = y / 2$$



Implementarea operatiilor geometrice

Umplerea pixelilor (*pixel filling*)

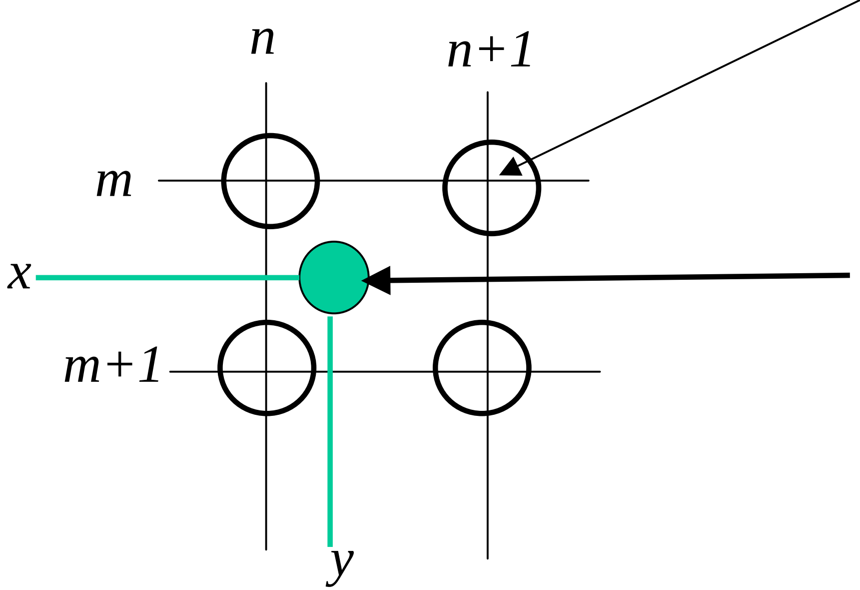


(x, y) coordonate ne-intregi inainte de transformare

(m', n') coordonate intregi dupa transformare

valoarea (inexistenta in imaginea reala initiala) trebuie obtinuta prin **interpolare** din valorile pixelilor vecini.

Metode de interpolare



pozitii de coordonate intregi in care sunt cunoscute valorile imaginii *f(m, n)*, *f(m+1, n)*, ...

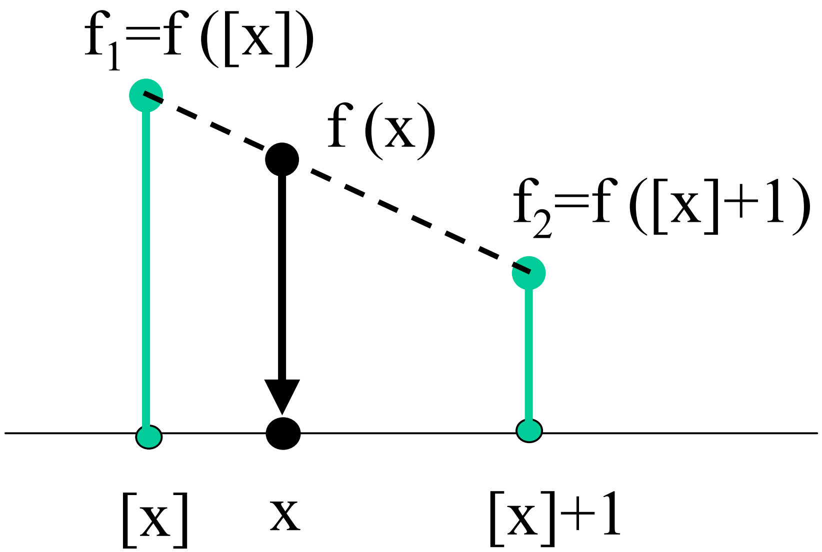
pozitie de coordonate ne-intregi in care se doreste calcularea valorii imaginii, presupusa continua pe domeniul *[m, m+1]* x *[n, n+1]*

Cel mai simplu: interpolare de ordinul 0

Valoarea in *(x,y)* este valoarea din cel mai apropiat punct de coordonate intregi – aceasta seamana cu o operatie punctuala.

Interpolarea biliniara

Interpolare liniara (de ordinul 1) dupa fiecare dimensiune.



$$f(x)=\frac{f([x]+1)-f([x])}{([x]+1)-[x]}\left(x-[x]\right)+f([x])$$

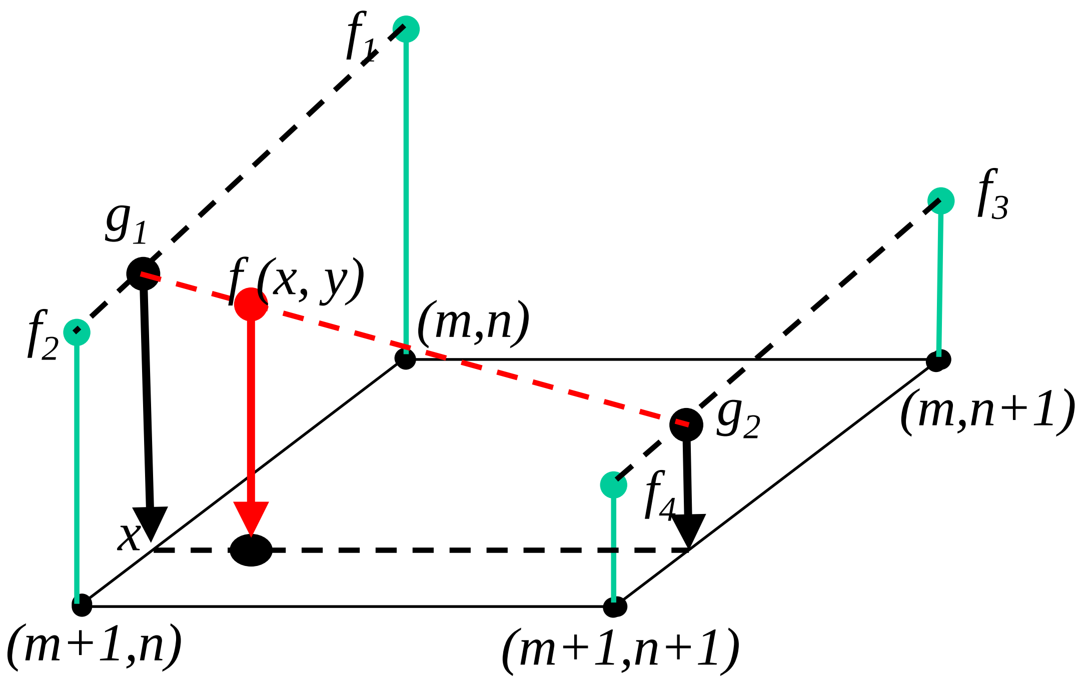
$$f(x)=(f_2-f_1)(x-[x])+f_1$$

Interpolarea biliniara

$f_1 = f(m, n)$
$f_2 = f(m+1, n)$
$f_3 = f(m, n+1)$
$f_4 = f(m+1, n+1)$

Deja este o prelucrarea de vecinatate !

$m = [x]$
$n = [y]$

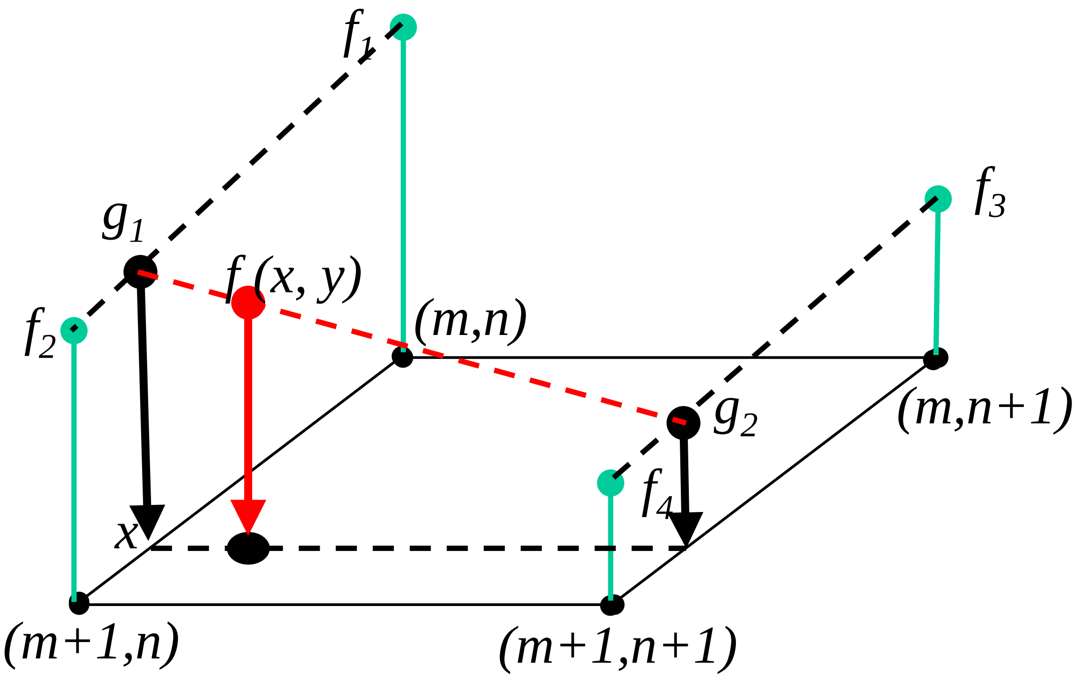


g_1 este interpolat liniar din f_1 si f_2

g_2 este interpolat liniar din f_3 si f_4

f este interpolat liniar din g_1 si g_2

Interpolarea biliniara



$f_1 = f(m, n)$
$f_2 = f(m+1, n)$
$f_3 = f(m, n+1)$
$f_4 = f(m+1, n+1)$

$m = [x]$
$n = [y]$

g_1 este interpolat liniar din f_1 si f_2

g_2 este interpolat liniar din f_3 si f_4

f este interpolat liniar din g_1 si g_2

$$g_1 = (f_2 - f_1)(x - m) + f_1$$
$$g_2 = (f_4 - f_3)(x - m) + f_3$$
$$f = (g_2 - g_1)(y - n) + g_1$$

Interpolarea biliniara

$$g_1 = (f_2 - f_1)(x - m) + f_1$$
$$g_2 = (f_4 - f_3)(x - m) + f_3$$
$$f = (g_2 - g_1)(y - n) + g_1$$

$$f = \left((f_4 - f_3)(x - m) + f_3 - (f_2 - f_1)(x - m) - f_1\right)(y - n) +$$
$$+ (f_2 - f_1)(x - m) + f_1$$

$$f = \left(f_4 - f_3 - f_2 + f_1\right)\left(x - m\right)(y - n) + (f_2 - f_1)(x - m) +$$
$$+ (f_3 - f_1)(y - n) + f_1$$

$$f = \alpha\left(x - m\right)(y - n) + \beta(x - m) + \gamma(y - n) + \delta$$

Metode de interpolare

Interpolari de ordin superior: 2, 3, ...

(functii spline – interpolari de ordin 3)