SEGMENTAREA IMAGINILOR

EXTRAGEREA CONTURURILOR

C. VERTAN

Segmentarea = descompunerea imaginii in partile sale componente.

(reducerea numarului de culori dintr-o imagine este un caz particular)

Segmentare :

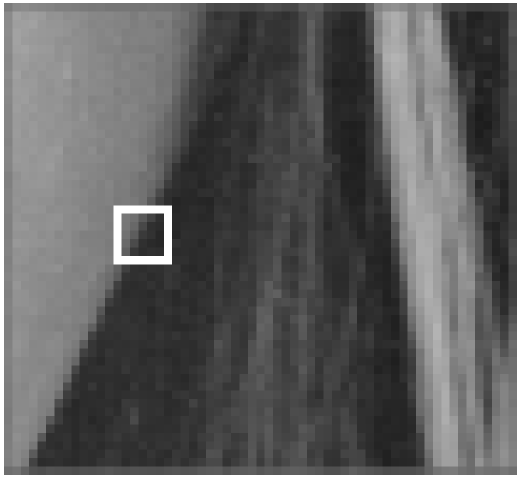
- orientata pe regiuni
- **orientata pe contururi**

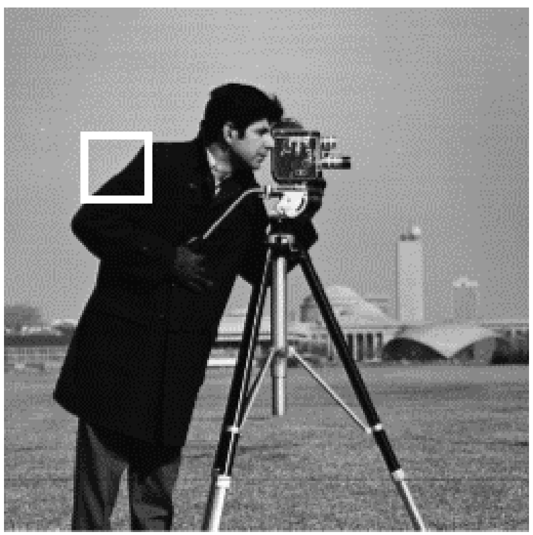
(abordari duale)

Contur:

margine a unei regiuni

zona de variatie puternica a valorilor pixelilor





Extragere contur = identificare a pixelilor in a caror vecinatate se produc variatii importante ale valorilor (nivelului de gri).

Marimea variatiei = intesitatea tranzitiei / conturului

Directia variatiei = directia perpendiculara tranzitiei / conturului

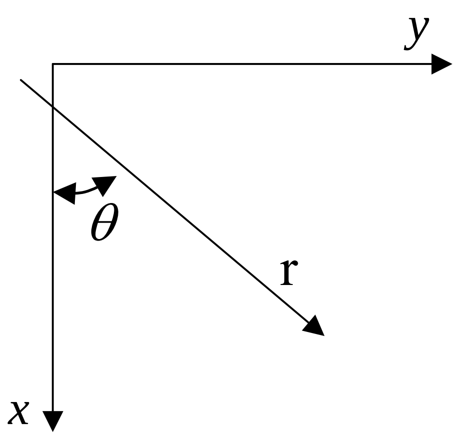
⇒ Extragerea conturului prin tehnici derivative (gradient al functiei imagine).

Tehnica de gradient

Fie imaginea f, modelata printr-o functie continua de doua variabile.

Derivata functiei imagine dupa o directie r oarecare este:

$$\frac{\partial f(x,y)}{\partial r}=\frac{\partial f(x,y)}{\partial x}\frac{\partial x}{\partial r}+\frac{\partial f(x,y)}{\partial y}\frac{\partial y}{\partial r}$$



$$\frac{\partial f(x,y)}{\partial r}=\frac{\partial f(x,y)}{\partial x}\cos\theta+\frac{\partial f(x,y)}{\partial y}\sin\theta$$

$$\frac{\partial f(x,y)}{\partial r}=f_x\cos\theta+f_y\sin\theta=F(\theta)$$

derivate dupa directii ortogonale fixate

Tehnica de gradient

Ceea ce intereseaza este :

directia r dupa care derivata este maxima
(pe ce directie este tranzitia cea mai puternica)
valoarea maxima a acestei derivate
(cat de puternica este tranzitia)

$$\frac{\partial f(x,y)}{\partial r} = f_x \cos\theta + f_y \sin\theta = F(\theta)$$

Trebuie studiata variatia acestei expresii in functie de θ.

$$\frac{\partial}{\partial\theta}\left(\frac{\partial f(x,y)}{\partial r}\right) = f_x \frac{\partial}{\partial\theta}\cos\theta + f_y \frac{\partial}{\partial\theta}\sin\theta = -f_x \sin\theta + f_y \cos\theta$$

Tehnica de gradient

$$\left.\begin{aligned}\frac{\partial}{\partial\theta}\left(\frac{\partial f(x,y)}{\partial r}\right) &= -f_x\sin\theta + f_y\cos\theta\\ \frac{\partial}{\partial\theta}\left(\frac{\partial f(x,y)}{\partial r}\right) &= 0\end{aligned}\right\} \quad \theta_0 = \arctan\frac{f_y}{f_x}$$

directia pe care apare variatia maxima.

Variatia maxima este :

$$\max\frac{\partial f(x,y)}{\partial r} = \left.\frac{\partial f(x,y)}{\partial r}\right|_{\theta=\theta_0} = \left. f_x\cos\theta + f_y\sin\theta\right|_{\theta=\theta_0} = \sqrt{f_x^2 + f_y^2}$$

Tehnica de gradient

Implementare:

calcul derivate verticala/ orizontala in fiecare pixel

calcul variatie maxima in fiecare pixel

daca variatia maxima intr-un pixel este suficient de mare, pixelul respectiv este pixel de contur

pentru pixelii de contur se calculeaza orientarea

Tehnica de gradient

Ce inseamna in discret derivarea pe directia orizontala/ verticala ?

Derivarea este o operatie liniara, deci se va implementa printr-un filtru liniar (definit de nucleul/ vecinatatea de filtrare).

Pentru o comportare derivativa suma ponderilor vecinatatilor este nula.

$$\begin{matrix} 0 & 0 & 0 \\ -1 & 0 & 1 \\ 0 & 0 & 0 \end{matrix} \qquad \begin{matrix} 0 & 0 & 0 \\ 1 & 0 & -1 \\ 0 & 0 & 0 \end{matrix} \qquad\qquad \begin{matrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & -1 & 0 \end{matrix} \qquad \begin{matrix} 0 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{matrix}$$

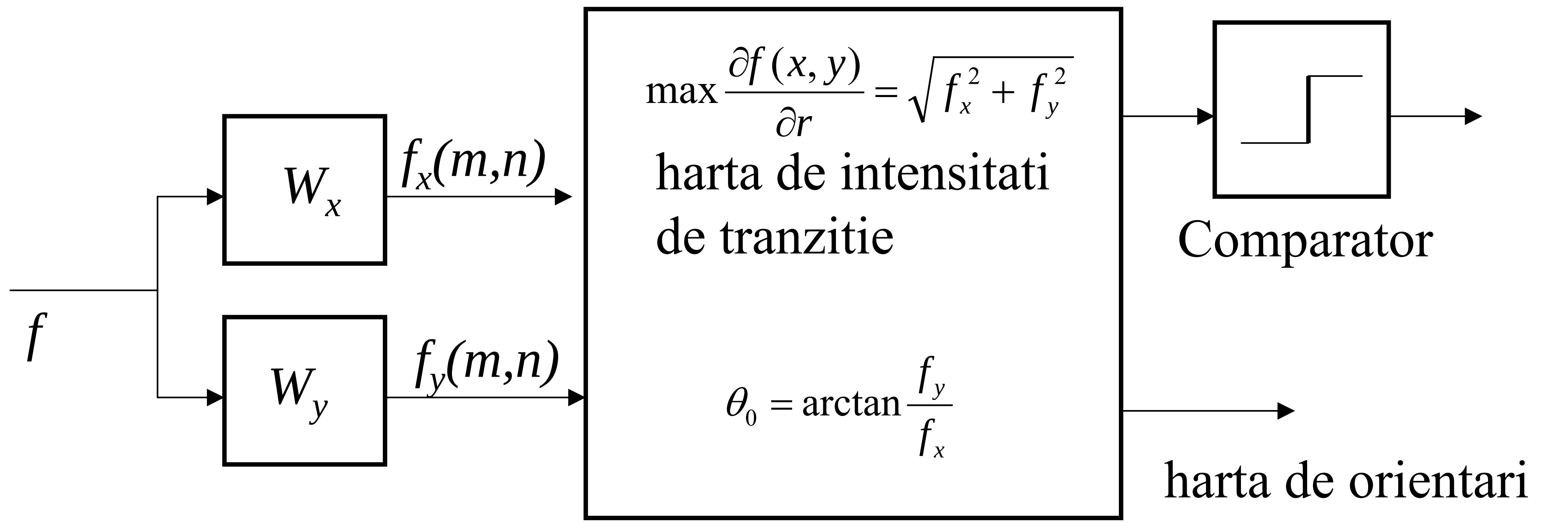
derivate pe directie orizontala

W_y

derivate pe directie verticala

W_x

Tehnica de gradient



Tehnica de gradient

Dezavantaj: derivata amplifica zgomotul.

Derivata trebuie combinata cu o filtrare de netezire, care sa o preceada.

Netezirea trebuie sa fie orientata, pe directie perpendiculara directiei de derivare.

$$\begin{matrix} 0 & 1 & 0 \\ 0 & c & 0 \\ 0 & 1 & 0 \end{matrix} \oplus \begin{matrix} 0 & 0 & 0 \\ -1 & 0 & 1 \\ 0 & 0 & 0 \end{matrix} = \begin{matrix} -1 & 0 & 1 \\ -c & 0 & c \\ -1 & 0 & 1 \end{matrix}$$

nuclee rotite cu 90°

$$\begin{matrix} 0 & 0 & 0 \\ 1 & c & 1 \\ 0 & 0 & 0 \end{matrix} \oplus \begin{matrix} 0 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{matrix} = \begin{matrix} -1 & -c & -1 \\ 0 & 0 & 0 \\ 1 & c & 1 \end{matrix}$$

Tehnica de gradient

Valori particulare pentru constanta de ponderare c:

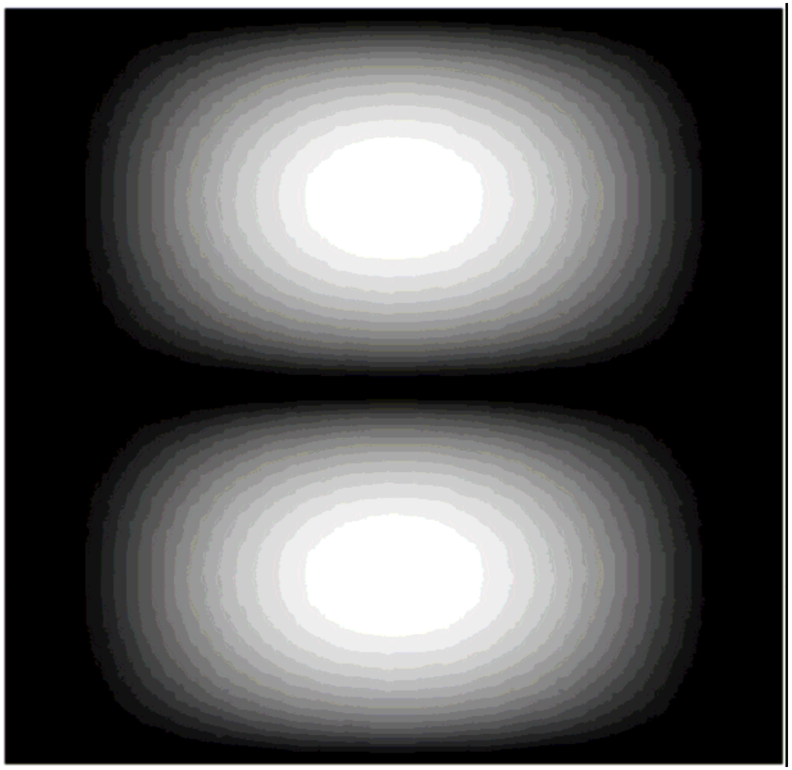
	W_y	W_x	
$c=1$	$\begin{matrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{matrix}$	$\begin{matrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{matrix}$	gradient Prewitt
$c=\sqrt{2}$	$\begin{matrix} -1 & 0 & 1 \\ -\sqrt{2} & 0 & \sqrt{2} \\ -1 & 0 & 1 \end{matrix}$	$\begin{matrix} -1 & -\sqrt{2} & -1 \\ 0 & 0 & 0 \\ 1 & \sqrt{2} & 1 \end{matrix}$	gradient izotrop
$c=2$	$\begin{matrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{matrix}$	$\begin{matrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{matrix}$	gradient Sobel

In frecventa

Contururile corespund frecventelor inalta; filtrele de detectie a contururilor au comportare de tip trece-sus sau trece-banda

$$\begin{matrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{matrix}$$

Sobel



C. VERTAN

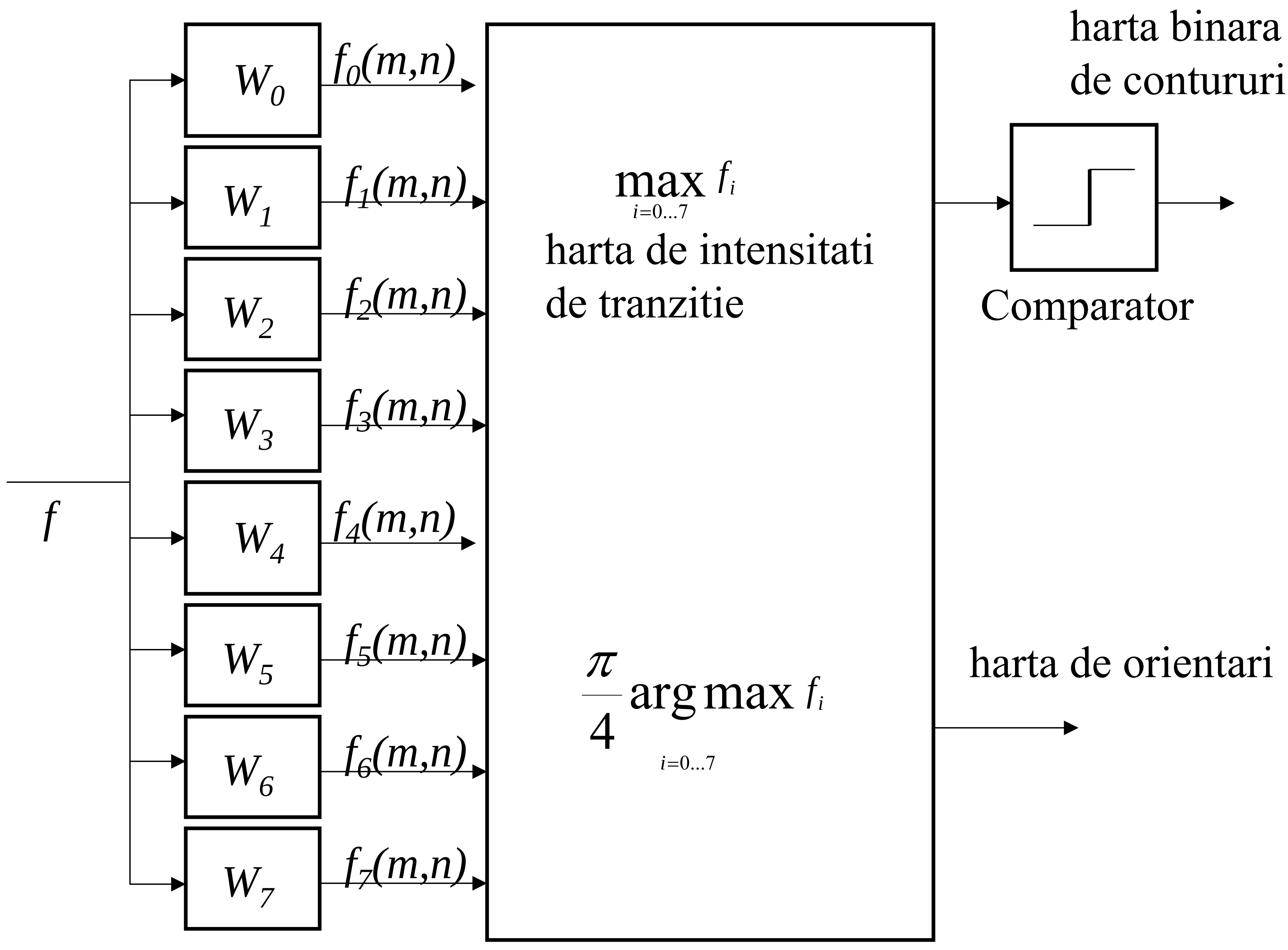
LABORATORUL DE ANALIZA ŞI PRELUCRAREA IMAGINILOR

Operatorul compas

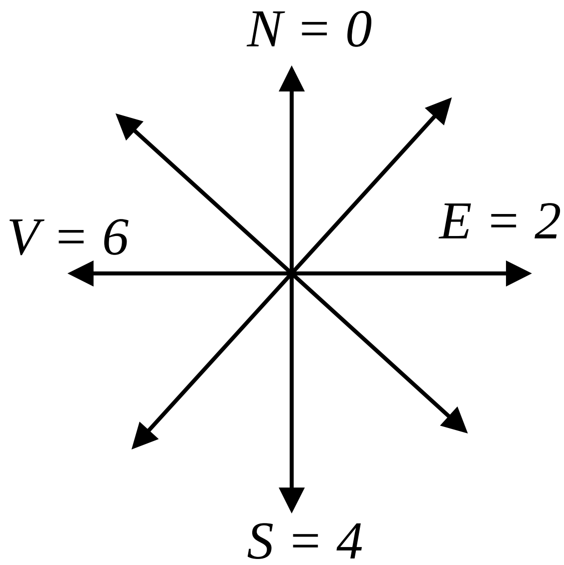
Nu toate orientarile sunt utile: precizia de reprezentare pe o grila discreta a unei drepte cu o orientare fixata este limitata.

Se pot reprezenta usor verticale, orizontale, diagonale.

De ce sa nu se calculeze intensitatea de variatie a imaginii numai dupa aceste directii ?


harta binara
de contururi
W_0
W_1
W_2
W_3
W_4
W_5
W_6
W_7
f
$f_0(m,n)$
$f_1(m,n)$
$f_2(m,n)$
$f_3(m,n)$
$f_4(m,n)$
$f_5(m,n)$
$f_6(m,n)$
$f_7(m,n)$
$\max_{i=0...7} f_i$
harta de intensitati
de tranzitie
$\frac{\pi}{4}\arg\max_{i=0...7} f_i$
Comparator
harta de orientari

Operatorul compas



Pornind de la o vecinatate de baza, restul vecinatatilor se obtin prin deplasari circulare ale frontierei vecinatatii cu o pozitie.

$$\begin{matrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{matrix} \qquad \begin{matrix} -2 & -1 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & 2 \end{matrix} \qquad \begin{matrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{matrix} \qquad \cdots\cdot$$

Sobel E   Sobel SE   Sobel S

Contururile extrase nu sunt “finale”: in general sunt groase si au puncte lipsa (inchiderea contururilor se face prin folosirea informatiei de orientare si reducerea pragului de definitie a pixelilor de contur).

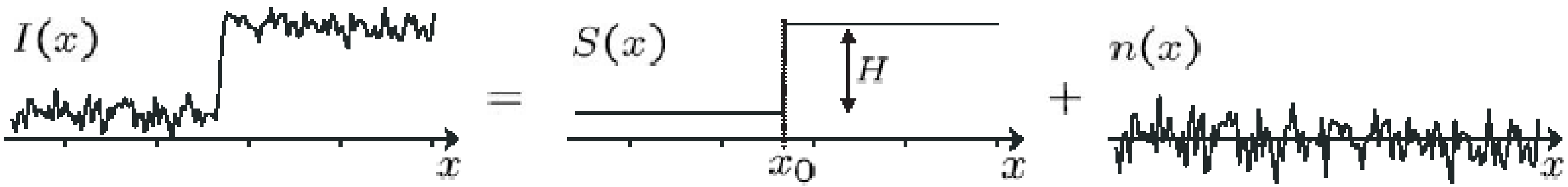
Exista extractoare “optimale” ale contururilor, in sensul pastrarii pozitiei spatiale a tranzitiei (localizare) si identificarii tranzitiilor lente (precizie) – filtrele Canny si Deriche.

Modelarea 1D

S(x) - muchie ideala de inaltime *H*

n(x) - zgomot alb, gaussian, aditiv

I(x) - muchie reala



$$I(x) = S(x) + n(x)$$

$$n(x) \sim N(0, \sigma_n^2) \quad , \qquad p_n(n; 0, \sigma_n^2) = \frac{1}{\sqrt{2\pi}\sigma_n} e^{-n^2/\sigma_n^2}$$

Filtrul liniar cautat : *f*

$$r(x) = f(x) * I(x) = f(x) * S(x) + f(x) * n(x)$$
$$= r_S(x) + r_n(x)$$

Pentru implementarea discreta a filtrului, folosind o fereastra centrata de dimensiune *K*, avem:

$$r_n(x) = \sum_{k=-K}^{K} f(-k)\, n(x+k)\,,$$

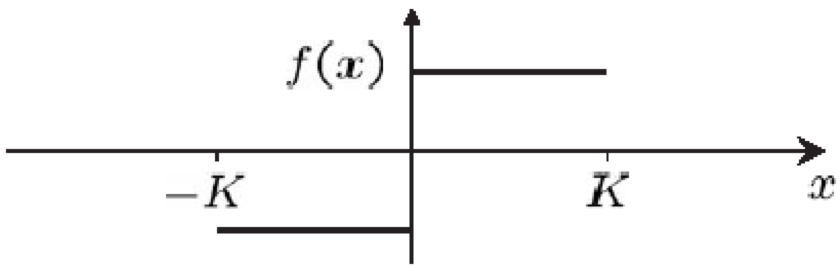
$$\mathrm{E}[r_n(x)] = \sum f(-k)\,\mathrm{E}[n(x+k)] = 0$$

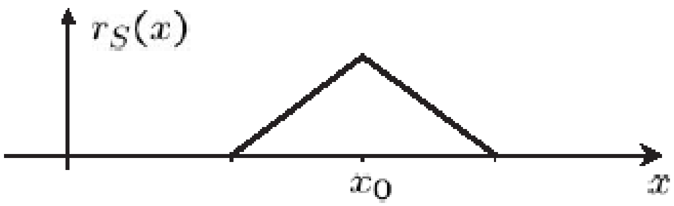
$$\mathrm{E}[r_n^2(x)] = \sum_k \sum_l f(-l)\, f(-k)\,\mathrm{E}[n(x+k)n(x+l)] = \sigma_n^2 \sum_k f^2(k)$$

Raportul semnal-zgomot la o pozitie oareacare x_0 este:

$$SNR = \frac{|(f * S)(x_0)|}{\sigma_n \sqrt{\sum_k f^2(k)}}$$

Criteriul 1 de optimizat: maximizarea raportului semnal-zgomot in pozitia de tranzitie (muchia), cu constrangerea ca raspunsul pentru un semnal constant sa fie nul.





$$\sum f(x) = 0.$$

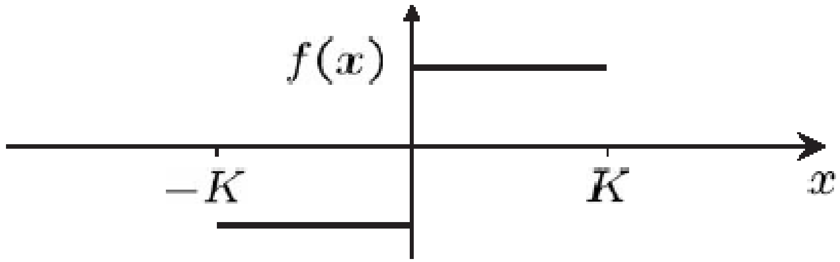
$$\sum f^2(x) = 1$$

Criteriul 2 de optimizat - localizarea buna: minimizeaza eroarea patratica medie dintre pozitia reala a muchiei si cel mai apropiat varf din raspunsul filtrului *r(x)*.

$$LOC = \frac{1}{\sqrt{\mathrm{E}[\min_k |x_l^* - x_0|^2]}}$$

x_l^* este pozitia unui maxim local in raspunsul filtrului.

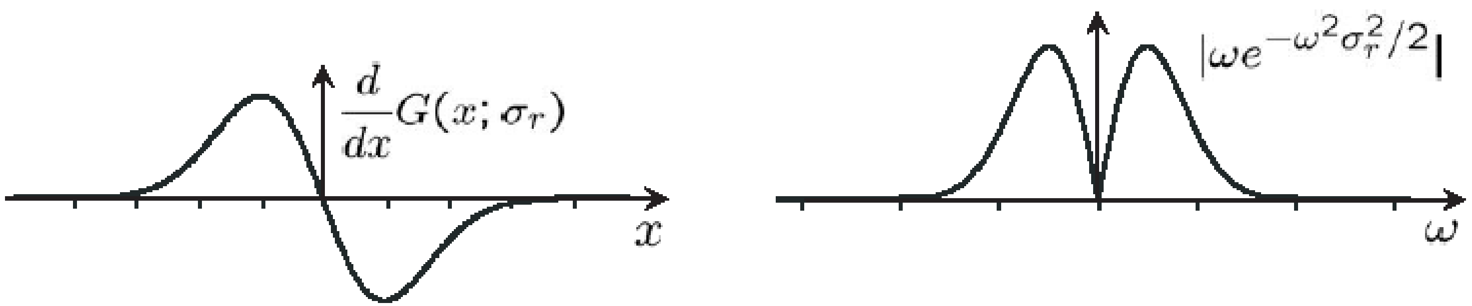
Maximizarea lui *SNR* x *LOC* produce acelasi filtru optim,



Criteriul 3 de optimizat - maxime rare: varfurile (maximele) din raspunsul filtrului *r(x)* trebuie sa fie, in medie, separate de cel putin o distanta *xPeak* (impusa).

$$\mathrm{E}[\,|x_{k+1}^* - x_k^*|\,] \;=\; xPeak$$

Filtrul optim depinde de raportul *xPeak/K.*
Pentru un raport mai mare ca 0.5, filtrul optim seamana cu o derivata de gaussiana.



$$f(x) \;\approx\; \frac{dG(x;\sigma_r)}{dx} \;=\; \frac{-x}{\sqrt{2\pi}\sigma_r^3}\, e^{-\frac{x^2}{2\sigma_r^2}} \qquad \text{cu} \qquad \mathcal{F}\left[\frac{dG(x;\sigma_r)}{dx}\right] \;=\; i\,\omega\, e^{-\frac{\omega^2\sigma_r^2}{2}}$$

Criteriul 3 este important, intrucat el stabileste parametrul de proiectare al filtrului: largimea de banda = dispersia gaussienei de baza din care provine filtrul optim.

Cu cat dispersia σ_r creste, detectia (*SNR*) se imbunatateste si localizarea (*LOC*) devine mai putin precisa.

Canny 2D

Ca si la tehnica simpla de gradient, extragerea contururilor se bazeaza pe amplitudinea derivatei directionale in directia perpendiculara conturului local.

normala: $\vec{\mathbf{n}} = (\cos\theta, \sin\theta)$

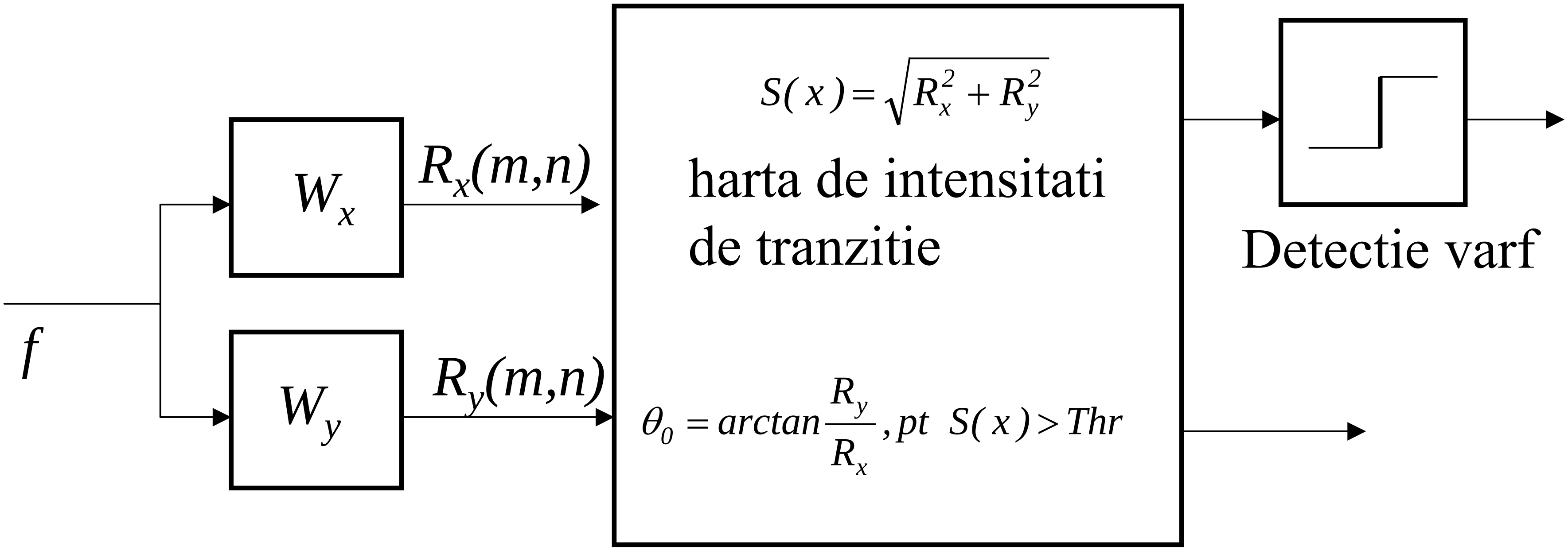
gaussiana: $G(\vec{\mathbf{x}};\ \sigma^2) \equiv \frac{1}{2\pi\sigma^2} e^{\frac{-(x^2+y^2)}{2\sigma^2}}$

derivata directionala:
$$\frac{\partial}{\partial\vec{\mathbf{n}}} G(\vec{\mathbf{x}};\ \sigma^2) = \nabla G(\vec{\mathbf{x}};\ \sigma^2) \cdot \vec{\mathbf{n}}$$
$$= \cos\theta\, G_x(\vec{\mathbf{x}};\ \sigma^2) + \sin\theta\, G_y(\vec{\mathbf{x}};\ \sigma^2)$$

Directia tranzitiei celei mai abrupte e data tot de gradient:

$$\vec{\mathrm{R}}(\vec{\mathbf{x}}) = \nabla G(\vec{\mathbf{x}};\ \sigma^2) * I(\vec{\mathbf{x}})$$

Implementare Canny 2D



Detectie varf = suprimare non-maxime in directia normalei, multi-scala si praguire cu histerezis de-a lungul conturului.

Observatii

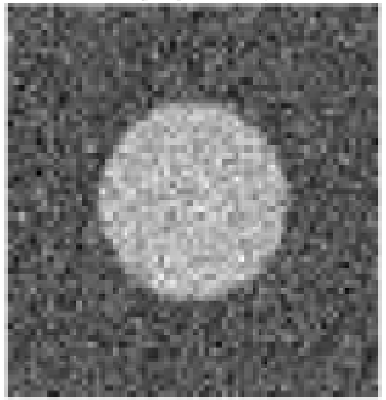
Derivata partiala a gaussianei pe o directie

$$\frac{\partial}{\partial x} G(\vec{\mathbf{x}};\, \sigma^2) \;=\; -\frac{x}{\sigma^2} G(x;\, \sigma^2)\, G(y;\, \sigma^2)$$

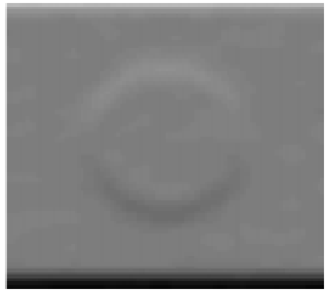
Dimensiunea suportului filtrului

$$K \geq 3\sigma_r.$$

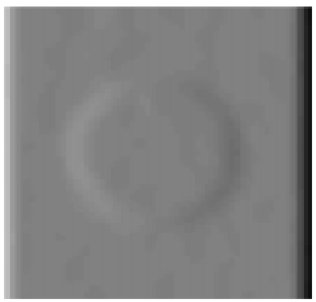
de obicei $K = 7, 9, 11$ pentru $\sigma_r = 1, 4/3, 5/3$



I

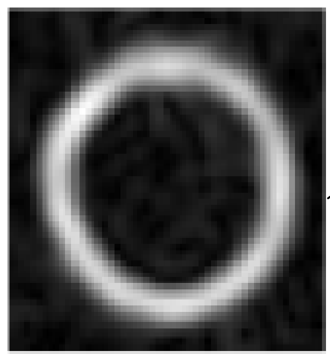


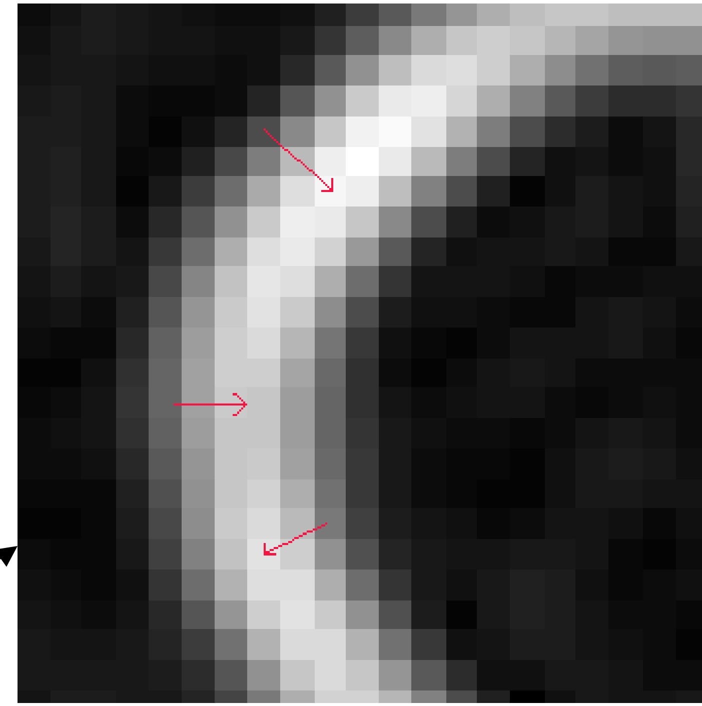
R_x



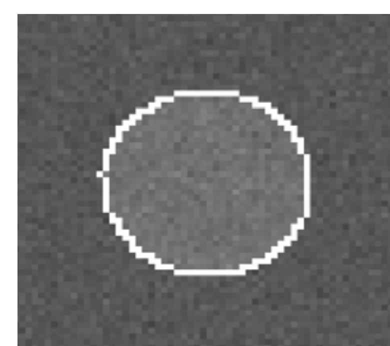
R_y

S

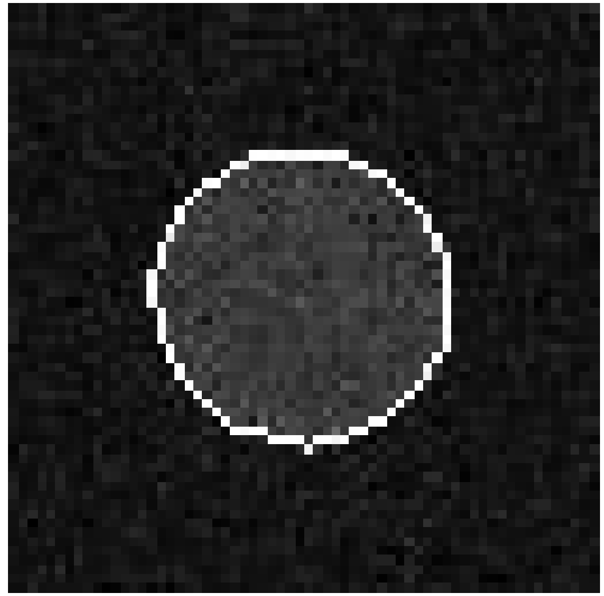


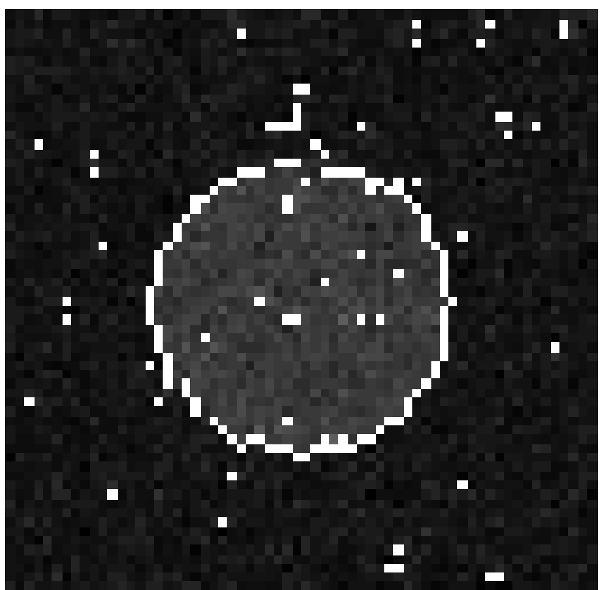


eliminarea valorilor ne-maximale
pe dir. perpendiculara conturului.

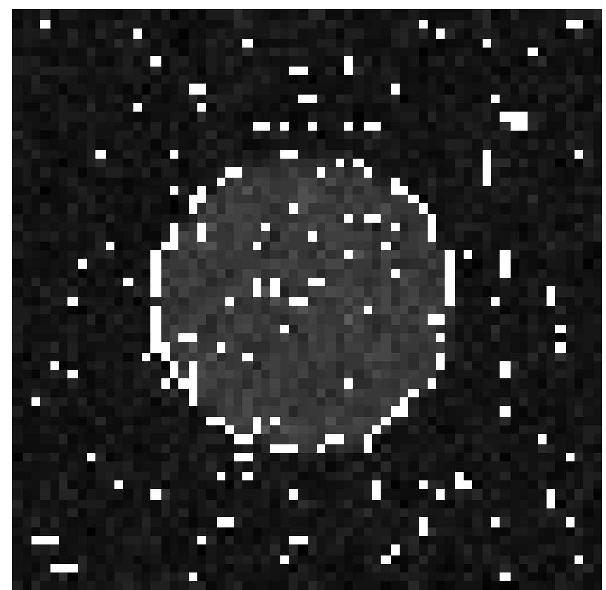


Canny





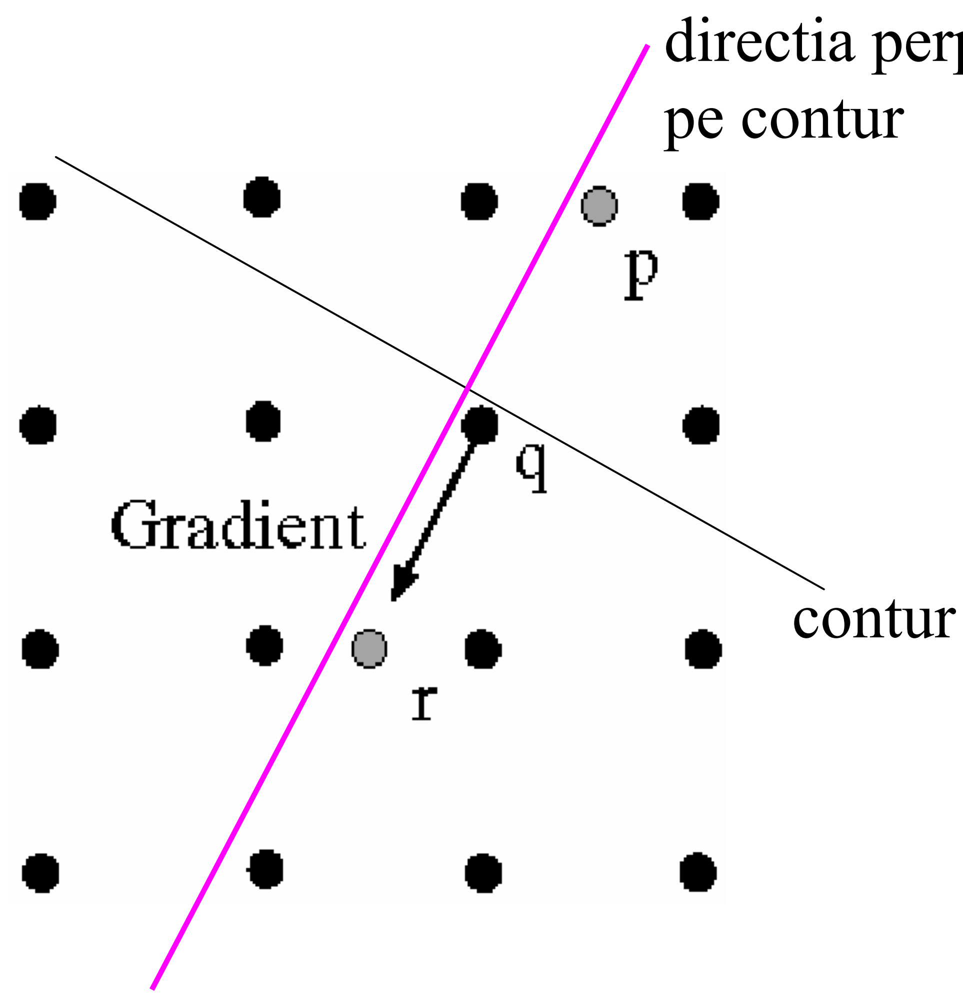
Sobel, [1 2 1; 0 0 0; −1 −2 −1] and its transpose



Roberts, [1 0; 0 −1] and its transpose

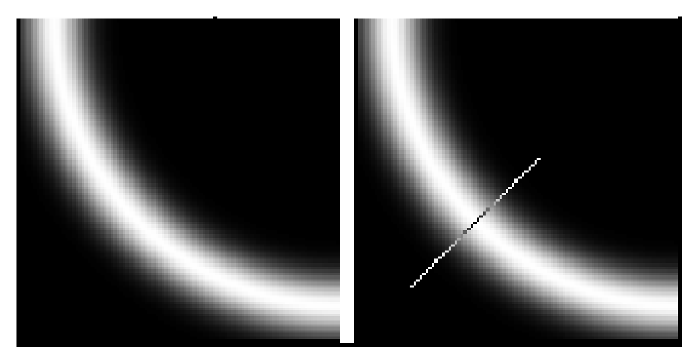
In practica

- suprimarea non-maximelor locale
- praguire cu histerezis



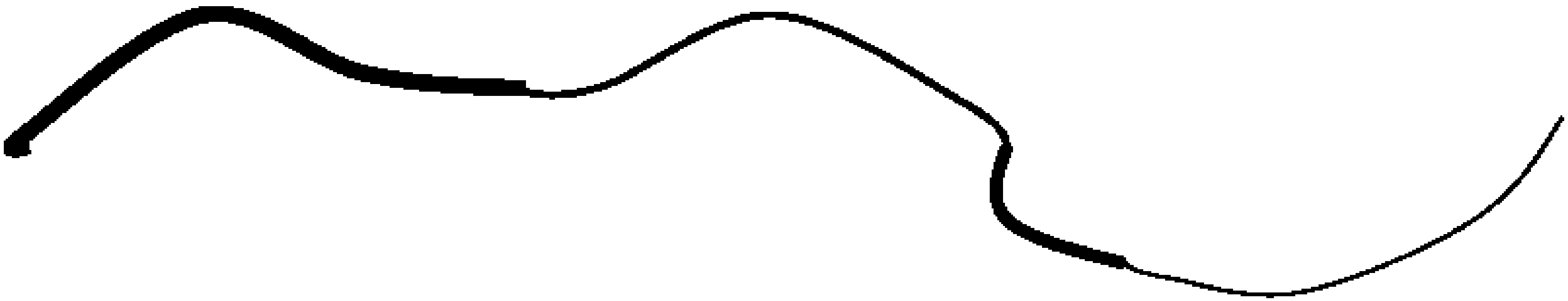
q este un punct pastrat ca maxim daca valoarea sa (gradientul imaginii in q) este mai mare decat valorile din punctele alaturate, r si p.

valorile acestea trebuiesc interpolate.



In practica - suprimarea non-maximelor locale
- praguire cu histerezis

Se folosesc doua praguri de selectie a punctelor de contur:
- un prag mare de selectie a unui punct sigur de contur (maxim in harta de intensitati de tranzitie sau gradient)
- un prag mic care selecteaza puncte din vecinatatea punctului de gradient mare - urmareste obtinerea de muchii continue chiar cand variatia gradientului muchiei este mica.



Metode neliniare

Gradientul morfologic

$$grad\, f = f \oplus b - f \ominus b == max - min$$

b = element structurant plat

Conturul interior/ exterior (morfologic)

$$\Delta f = f \oplus b - f$$

$$\delta f = f - f \ominus b$$

se poate arata ca, pentru un element structurant plat cu suport V_8, acesta este un operator compas

Metodele produc harti de intensitati de tranzitie, ce trebuie praguite….

element structurant plat
cu suport 5 x 5

contur exterior

contur interior

gradient morfologic

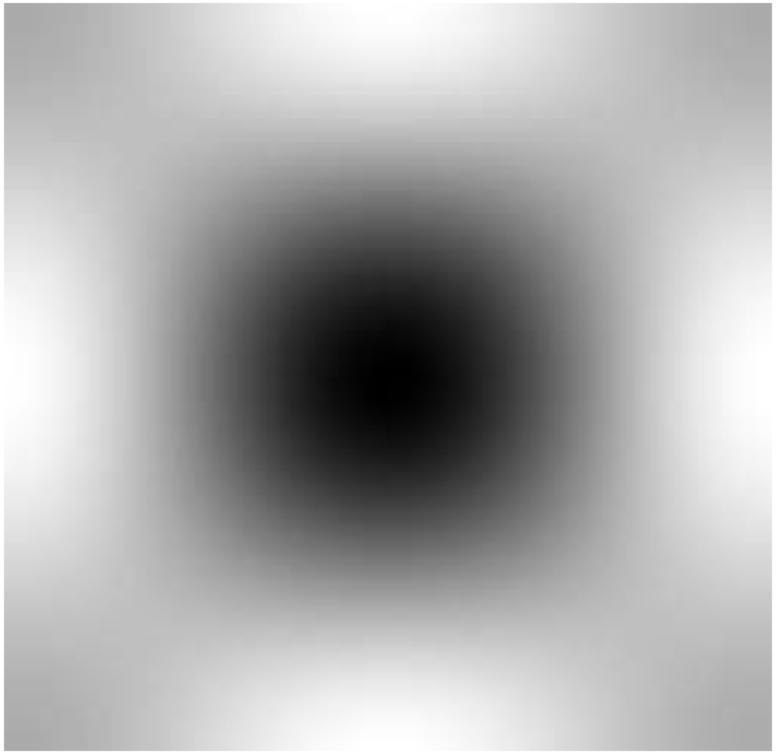
Folosirea unei derivate de ordinul 2 - laplacian

In frecventa

Contururile corespund frecventelor inalta; filtrele de detectie a contururilor au comportare de tip trece-sus sau trece-banda

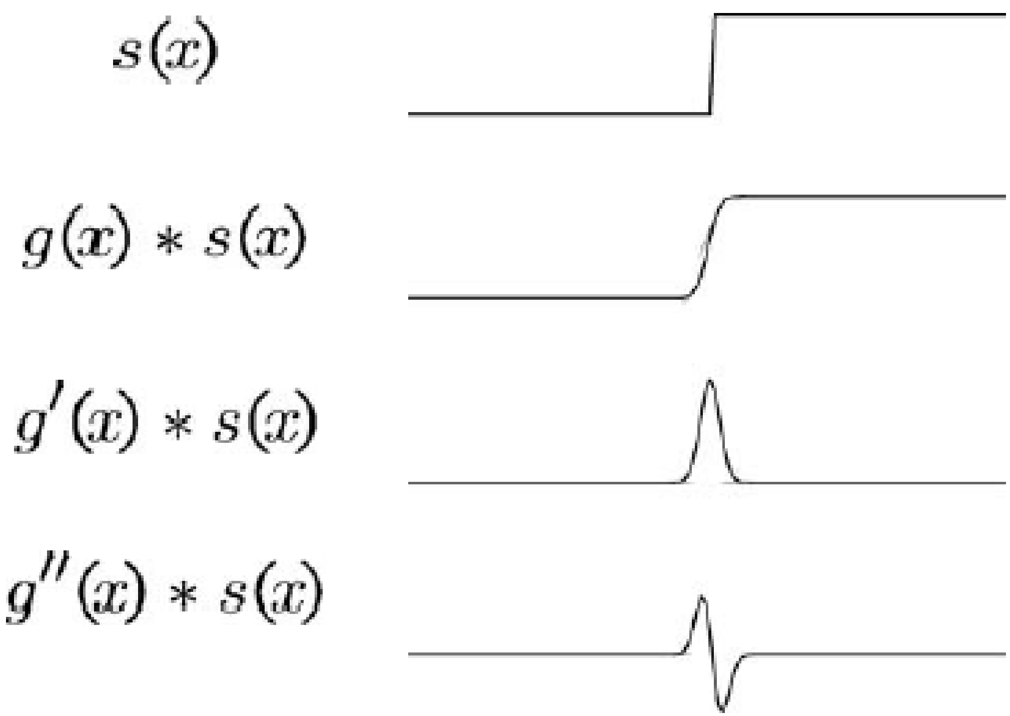
$$\begin{array}{ccc} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{array}$$

Laplacian



Zero-crossing (Marr)

Folosirea derivatei secunde pentru detectia contururilor.



Raspunsul maxim al derivatei intai este trecerea prin zero a derivatei a doua.

In practica se foloseste operatorul LoG - Laplacian of Gaussian (pt a include prop de netezire in operator).

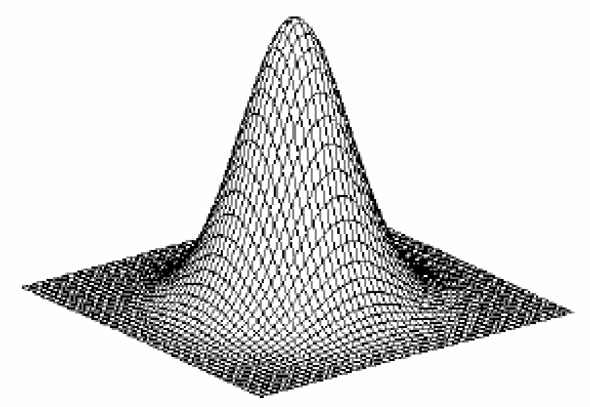
$$G(\vec{\mathbf{x}};\, \sigma^2) \equiv \frac{1}{2\pi\sigma^2} e^{\frac{-(x^2+y^2)}{2\sigma^2}}$$

$$\frac{\partial^2}{\partial x^2} G(x,\sigma^2) = \left(\frac{x^2}{\sigma^2} - 1 \right) \frac{1}{\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$

$$\frac{\partial^2}{\partial y^2} G(y,\sigma^2) = \left(\frac{y^2}{\sigma^2} - 1 \right) \frac{1}{\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$

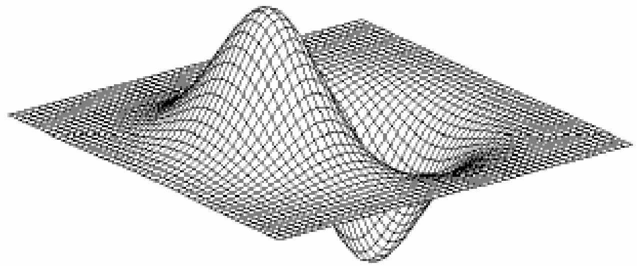
$$\nabla^2 G(x,y,\sigma^2) = \frac{1}{\sigma^2} \left(\frac{x^2+y^2}{\sigma^2} - 2 \right) e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$

Mai ramane problema determinarii trecerilor prin zero…



Gaussian

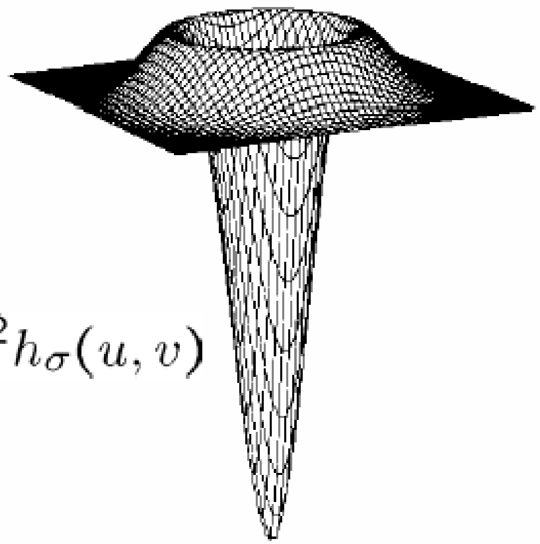
$$h_\sigma(u,v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{2\sigma^2}}$$



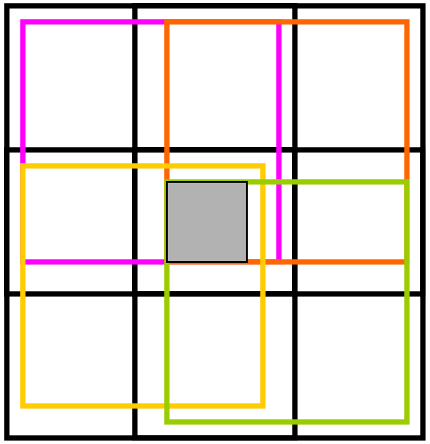
derivative of Gaussian

$$\frac{\partial}{\partial x} h_\sigma(u,v)$$

Laplacian of Gaussian



$$\nabla^2 h_\sigma(u,v)$$



Cvadrante de impartire a vecinatatii 3x3 a punctului curent verificat pt tranzitia prin zero.

Tranzitia prin zero inseamna ca intre 2 cvadrante valoarea derivatei isi schimba semnul.

Deci vom avea de determinat daca exista un cvadrant de valoare medie pozitiva si un cvadrant de valoare medie negativa.