PARAMETRI DE FORMA

Abordare regiune

C. VERTAN

LABORATORUL DE ANALIZA ŞI PRELUCRAREA IMAGINILOR

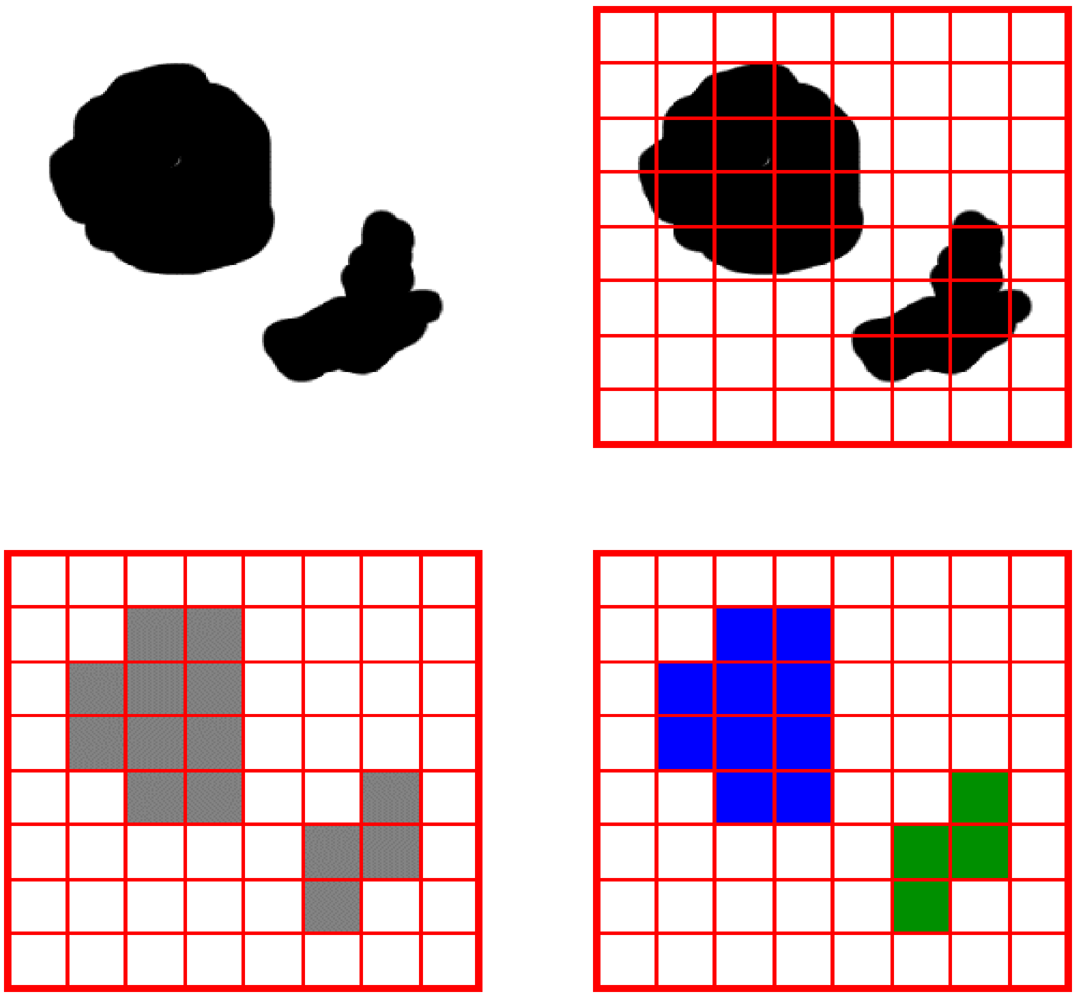
Parametri de forma

Asociaza unei forme (multime binara in planul 2D) un set de numere prin care aceasta poate fi recunoscuta, indiferent de pozitie, dimensiune, orientare.

PARAMETRI DE FORMA:

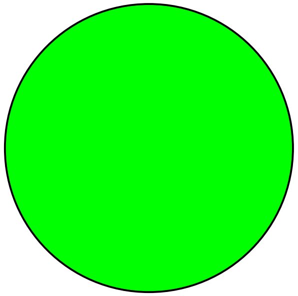
Rapoarte de aspect

Aria = numar de pixeli

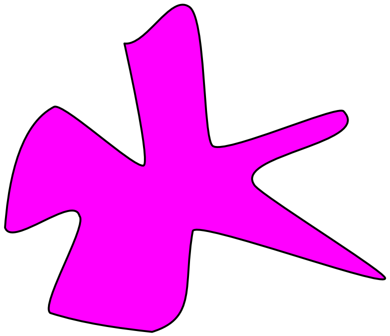


Compacitate

- P^2/A: perimetru x perimetru / arie
  - adimensional
  - minimal pentru disc
  - invariant la rotatie



compact



non compact

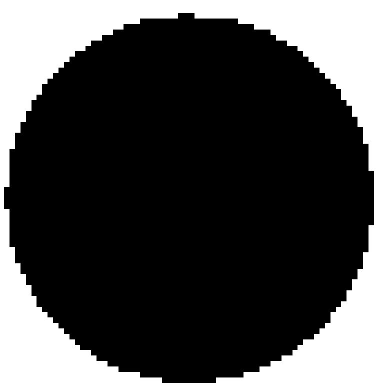
P^2/A

perimetru x perimetru / arie

normalizare: $$\frac{P^2}{4\pi A}$$

arie = 10538, perimetru = 798

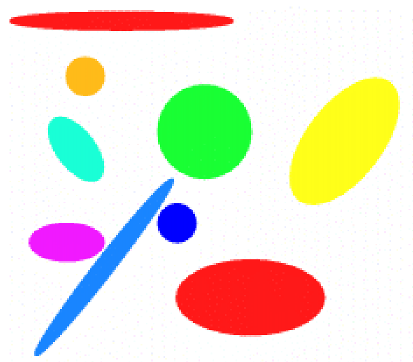
$P^2/A=60.43$, $P^2/A_{norm}=4.81$

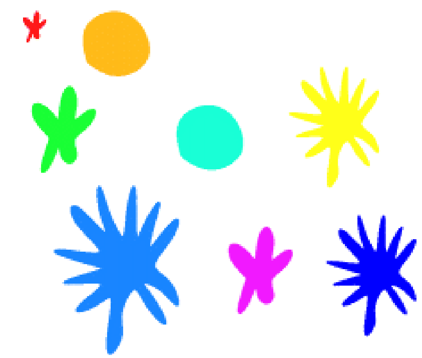


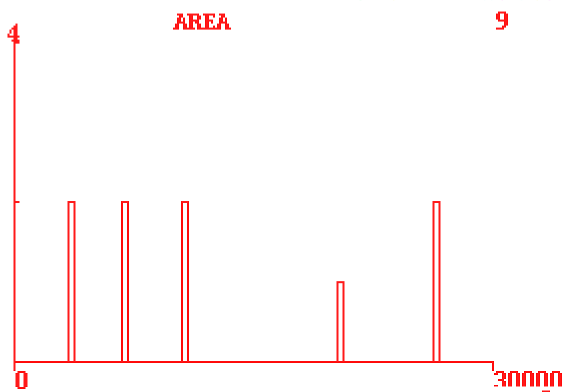
arie = 3591, perimetru = 221

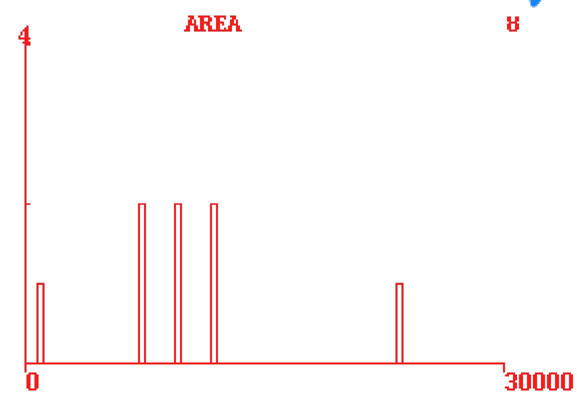
$P^2/A=13.60$, $P^2/A_{norm}=1.08$

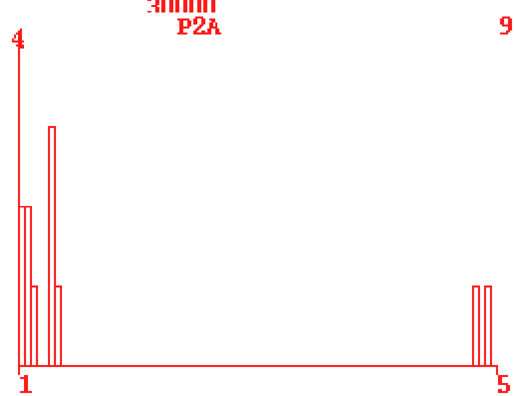
Exemple de masuratori P^2/A

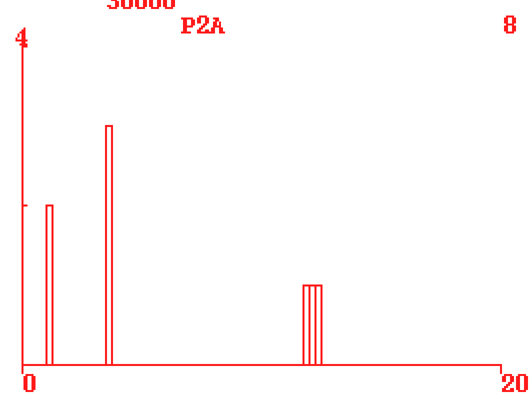






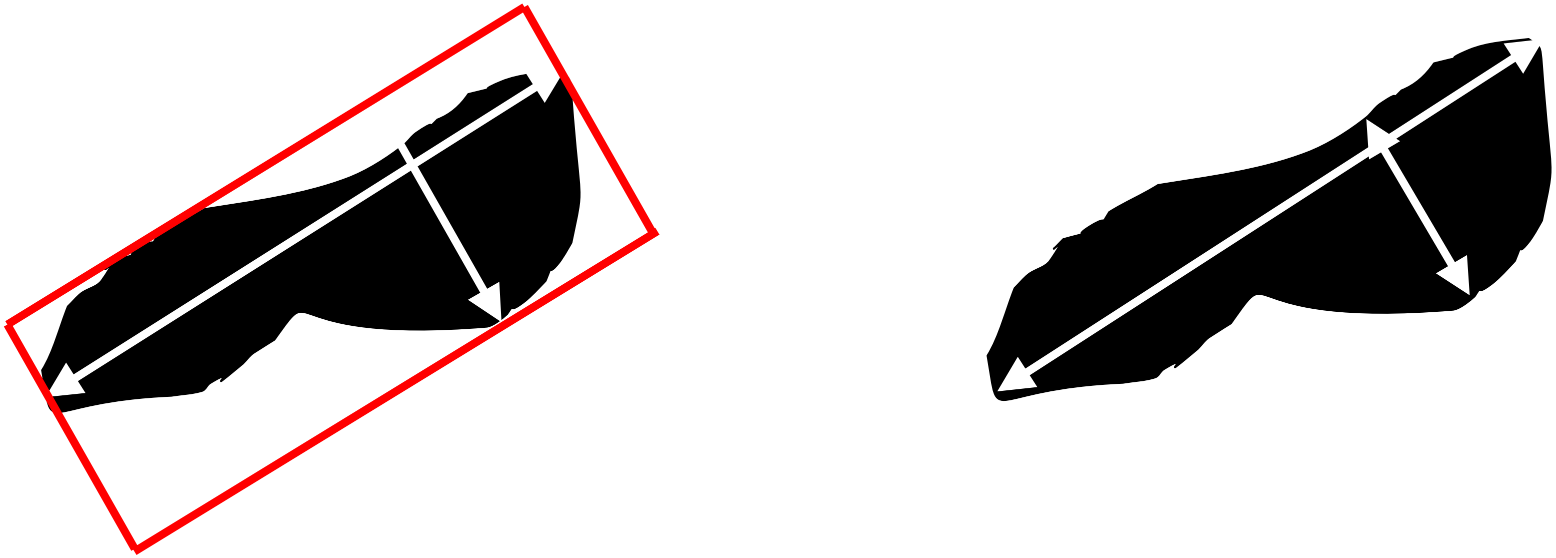






Excentricitate

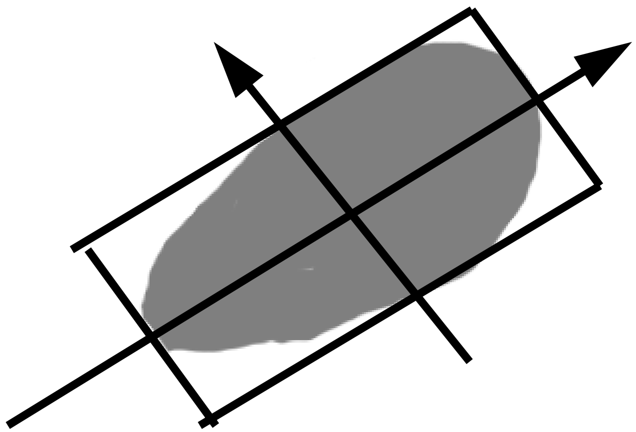
- cea mai lunga coarda/ coarda perpendiculara



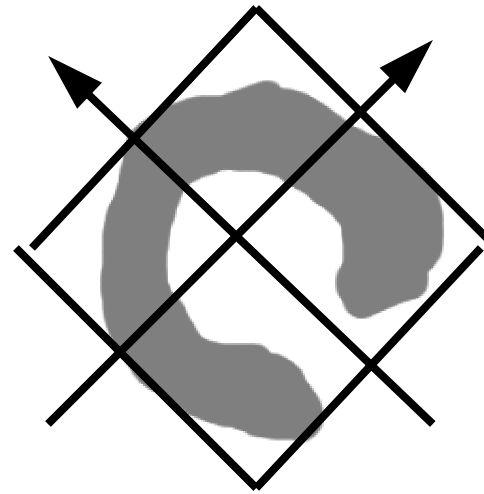
- exista definitii alternative!

Elongatie

1. raport dimensiuni pt dreptunghiul minim de incadrare



OK

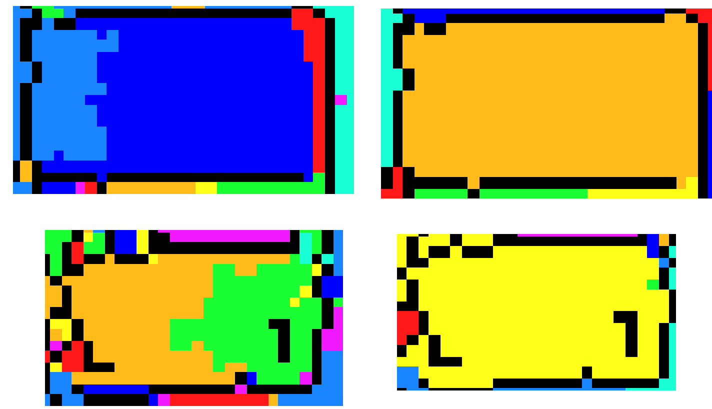


not OK

2. arie/($2d^2$)
   – d e latimea maxima
3. calea maxima

Rectangularitate

- aria regiunii/ aria dreptunghiului de incadrare



Topologie

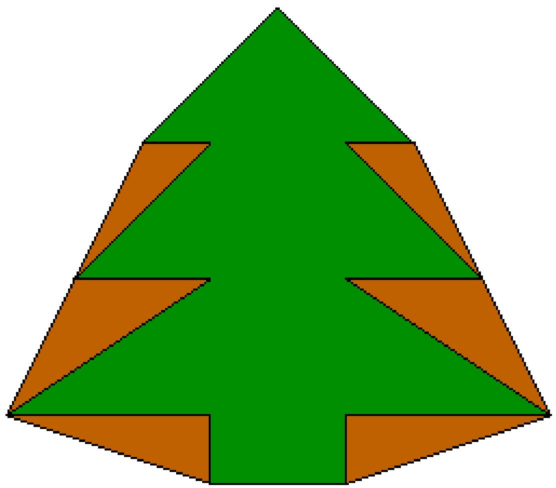
- numarul lui Euler

$$E=C-H$$

  - C – numar de componente conexe
  - H – numar de gauri

Anvelopa convexa

- regiune convexa:
  - pt. orice $x_1, x_2 \in R$, segmentul $[\ x_1\ x_2\ ]$ este in R
- anvelopa convexa (convex hull) CH(R)
  - cea mai mica multime convexa ce contine R
- deficitul de convexitate = CH(R) - R



PARAMETRI DE FORMA:

MOMENTE STATISTICE SI INVARIANTI

Momentele statistice ale unei forme descrise de functia binara *f*.

Forma este echivalenta cu suportul functiei (domeniul in care aceasta ia valori nenule), pe care valorile functiei sunt unitare.

$$m_{pq} = \iint_{Supp(f)} f(x,y) x^p y^q dxdy$$ coordonate continue

$$m_{pq} = \sum_{f(x,y)\neq 0} \sum x^p y^q$$ coordonate discrete

$p, q = 0, 1, 2, \ldots$

Aria

Moment statistic particular: m_{00}

$$m_{00} = \iint_{Supp(f)} f(x,y)dxdy = Arie(Supp(f))$$

$$m_{00} = \sum_{f(x,y)\neq 0} \sum 1 = Arie(Supp(f))$$

Momentele statistice nu prezinta nici un grad de invarianta. (depinde pozitia formei in imagine, de dimensiunea si orientarea acesteia).

Centrul de greutate

Coordonatele centrului de greutate al fomei, (μ_x, μ_y) se obtin prin:

$$\mu_x = \frac{m_{10}}{m_{00}}$$

$$\mu_y = \frac{m_{01}}{m_{00}}$$

Momente statistice centrate

asigura invarianta in raport cu translatia.

$$\mu_{pq} = \iint\limits_{Supp(f)} f(x,y)\left(x-\mu_x\right)^p\left(y-\mu_y\right)^q dxdy$$

$$\mu_{pq} = \sum_{f(x,y)\neq 0}\sum \left(x-\mu_x\right)^p\left(y-\mu_y\right)^q$$

Momente statistice centrate normalizate

asigura invarianta in raport cu translatia si scalarea.

$$\eta_{pq} = \frac{\mu_{pq}}{\mu_{00}^{(p+q+2)/2}}$$

Invariantii formei [Hu]

invarianti la translatie, scalare, rotatie, reflexie.

$$\Phi_1 = \eta_{20} + \eta_{02}$$

$$\Phi_2 = (\eta_{20} - \eta_{02})^2 + 4\eta_{11}^2$$

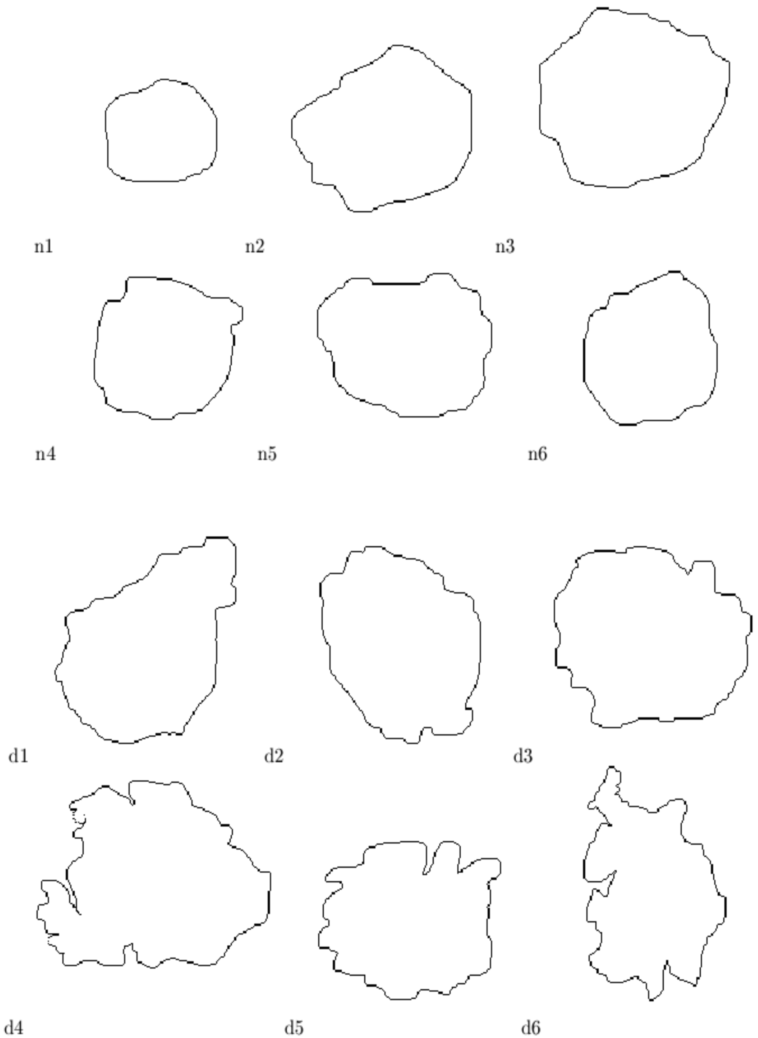
$$\Phi_3 = (\eta_{30} - 3\eta_{12})^2 + (\eta_{03} - 3\eta_{21})^2$$

$$\Phi_4 = (\eta_{30} + \eta_{12})^2 + (\eta_{03} + \eta_{21})^2$$

$$\Phi_5 = (\eta_{30} - 3\eta_{12})(\eta_{30} + \eta_{12})\left[(\eta_{30} + \eta_{12})^2 - 3(\eta_{03} + \eta_{21})^2\right]$$
$$+ (\eta_{03} - 3\eta_{21})(\eta_{03} + \eta_{21})\left[(\eta_{03} + \eta_{21})^2 - 3(\eta_{30} + \eta_{12})^2\right]$$

$$\Phi_6 = (\eta_{20} + \eta_{02})\left[(\eta_{30} + \eta_{12})^2 - (\eta_{03} + \eta_{21})^2\right] + 4\eta_{11}(\eta_{03} + \eta_{21})(\eta_{03} + \eta_{21})$$

Exemplu



n1,…,n6=normal

Formele difera prin dimensiune, orientare, iregularitate…

d1,…,d6=diabetic

Orientarea formei

Directia dupa care momentul de inertie al formei este minim.

$$\theta = \frac{1}{2}\arctan\frac{2\mu_{11}}{\mu_{20}-\mu_{02}}$$

PARAMETRI DE FORMA:

Aproximari morfologice ale formei: Skeletonul morfologic

C. VERTAN

Nume echivalent : MAT - Median Axis Transform
- modelul focului in preerie

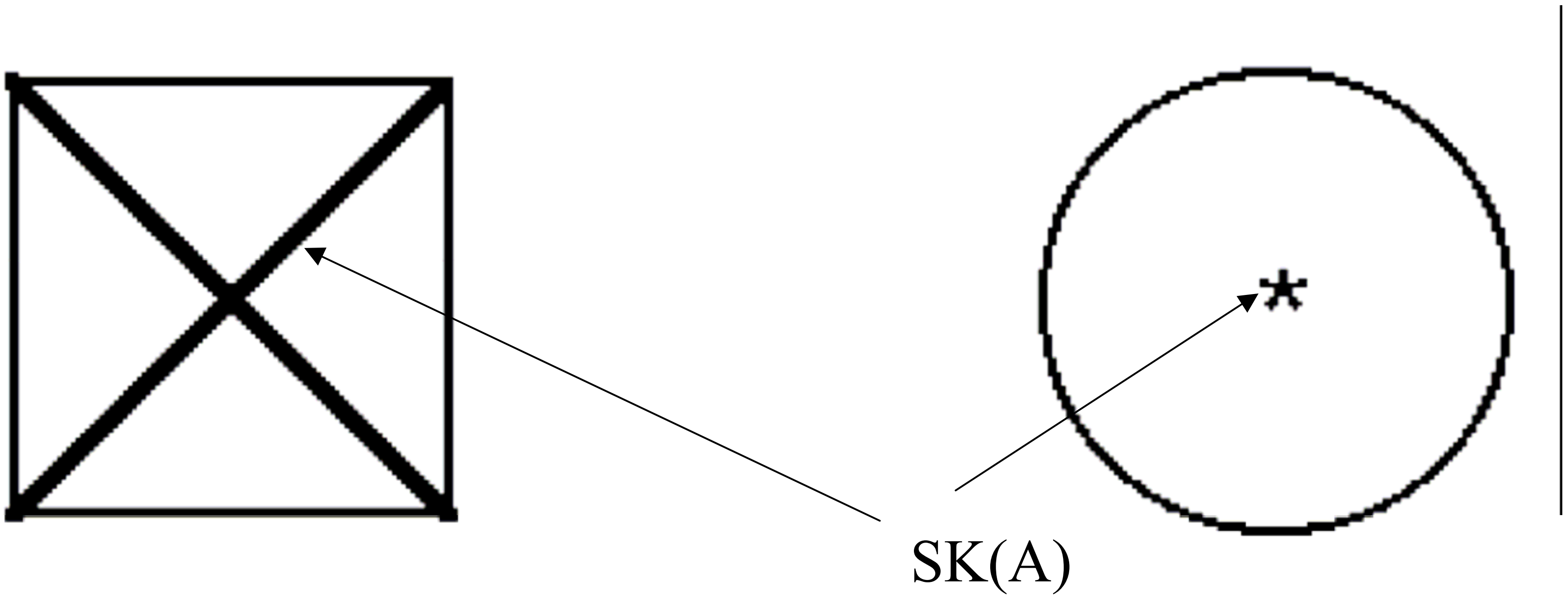
Se defineste pe baza conceptului de **disc maximal** intr-o forma A

$B_{\mathbf{x}}(r)$ | disc de centru **x** si raza *r*

$$B_{\mathbf{x}}(r) \subseteq A$$

$$B_{\mathbf{x}}(r) \subseteq B_{\mathbf{x}'}(r') \subseteq A \Leftrightarrow \begin{cases} r = r' \\ \mathbf{x} = \mathbf{x}' \end{cases}$$

Skeletonul unei forme este multimea centrelor discurilor maximal in forma.



Cum se implementeaza in cazul discret, cu operatori morfologici ?

Skeletonul morfologic

$$SK(A) = \bigcup_{n=0}^{N_{\max}} S_n(A)$$

$$S_n(A) = (A \ominus nB) - (A \ominus nB) \circ B$$

$$nB = B \oplus \ldots \oplus B \ (\text{de } n \text{ ori})$$

B este elementul structurant ales (imagine a discului unitar)

Skeletonul morfologic : reconstructia

$$A = \bigcup_{n=0}^{N_{\max}} S_n(A) \oplus nB$$

Skeletonul morfologic : aproximarea formei

$$\widetilde{A}_k = \bigcup_{n=k}^{N_{\max}} S_n(A) \oplus nB$$

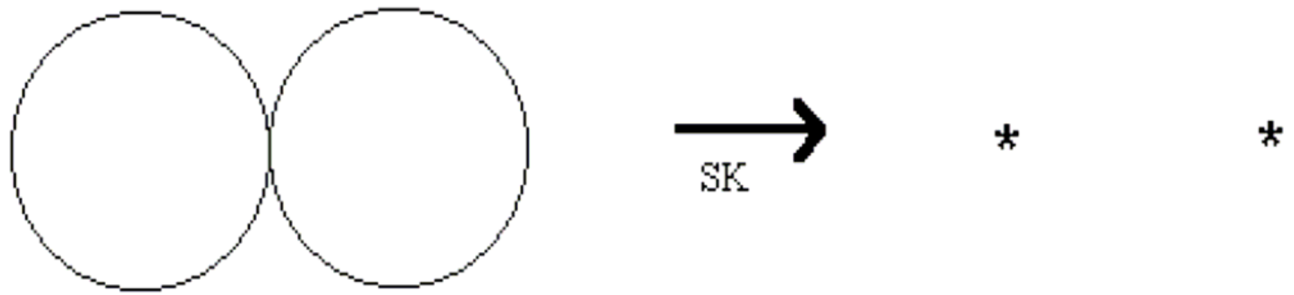
compresie/ reconstructie multirezolutie

Skeletonul morfologic : alte proprietati

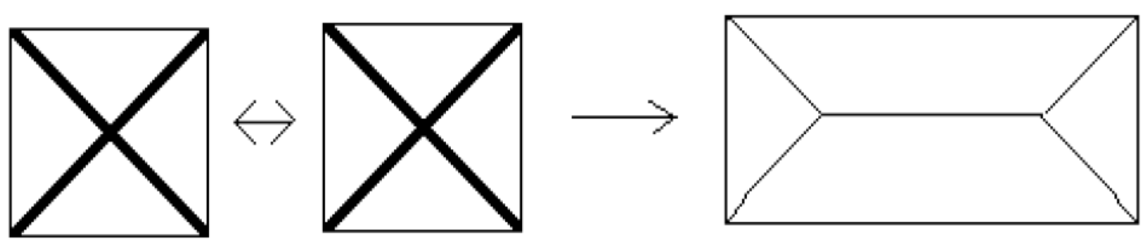
$$SK(SK(A)) = SK(A)$$ (idempotenta)

$$S_i(A) \cap S_j(A) = \emptyset \quad \forall i \neq j$$

nu pastreaza
conexitatea



nu comuta
cu reuniunea



slaba rezistenta la zgomot

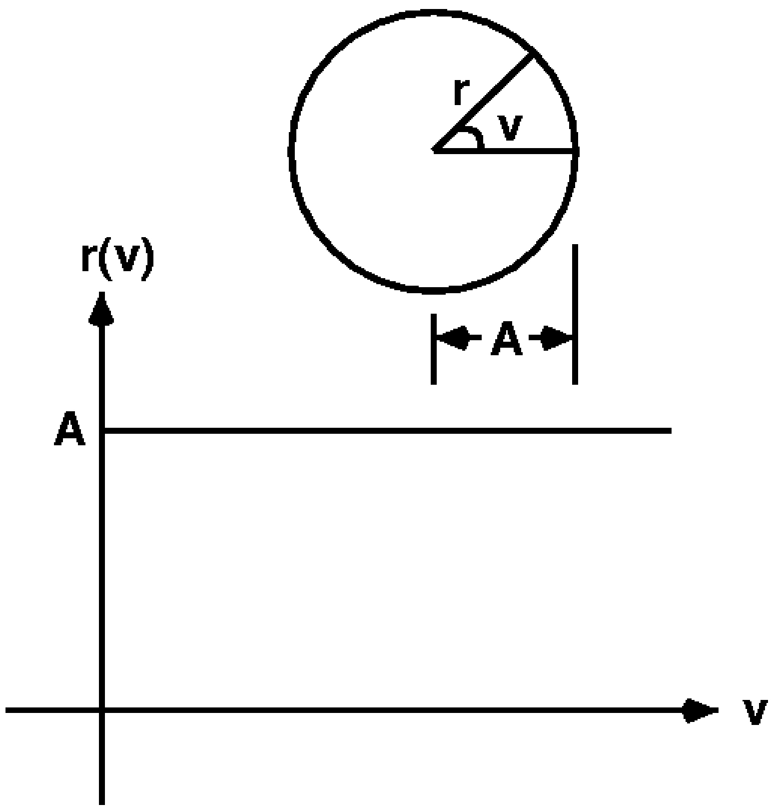
PARAMETRI DE FORMA:

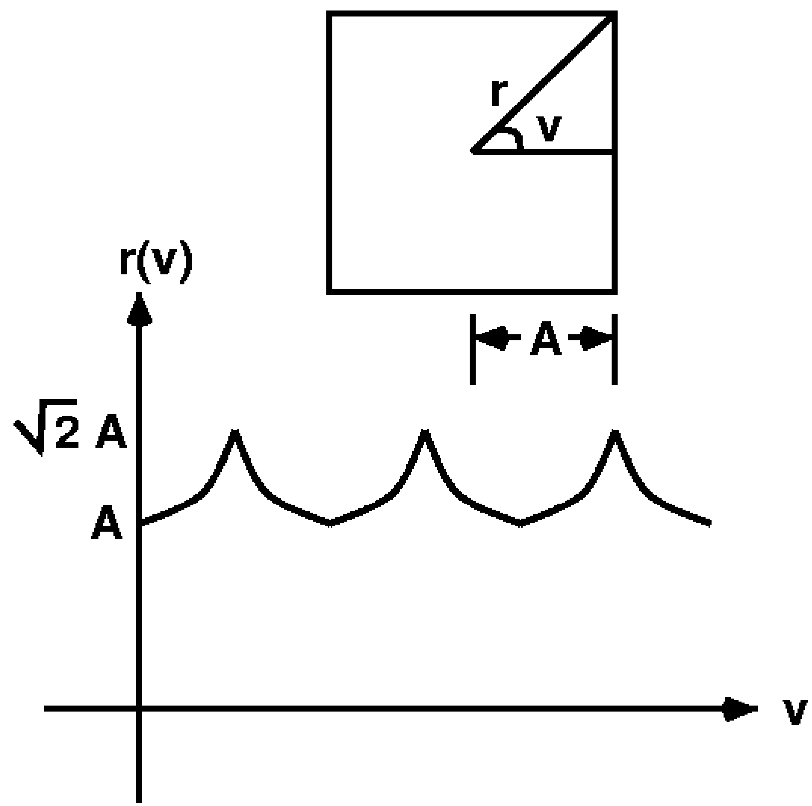
DESCRIEREA CONTURURILOR

Semnatura formei

- reprezentare functionla 1D a conturului
- abordare simpla: distanta de la un punct de referinta (de obicei centrul de greutate) ca functie de unghiul la centru
- frontiera 2D $\Rightarrow$ functie 1D
- probleme la rotatie si scalare
  - selectie centru
  - selectie punct de start
  - rescalare functie, de ex. valori $\in [0,1]$

Exemple de semnaturi





Descriptori Fourier de contur

- frontiera de K pixeli e reprezentata ca o secventa de coordonate
  - s(k)=(x(k),y(k)), k=0,1,2,...,K-1
  - numar complex s(k)=x(k)+iy(k)
  - 2D→1D
- DFT

$$a(u)=\frac{1}{K}\sum_{k=0}^{K-1}s(k)e^{-j2\pi uk/K},u=0,1,2,\ldots,K-1$$

a(u) – descriptorii Fourier ai frontierei

Reconstructia formei din descriptorii Fourier

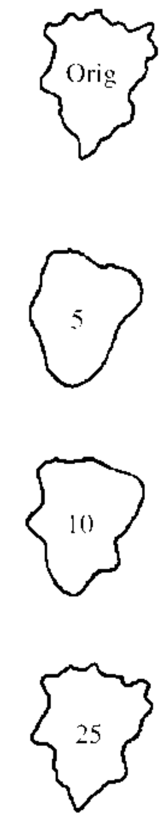
- reconstructie = DFT invers

$$s(k)=\sum_{u=0}^{K-1} a(u)e^{j2\pi uk/K}, k=0,1,2,\ldots,K-1$$

- aproximarea frontierei daca se foloseste o serie truncheata de coeficienti (P<K)

$$\hat{s}(k)=\sum_{u=0}^{P-1} a(u)e^{j2\pi uk/K}, k=0,1,2,\ldots,K-1$$

  - frontiera va avea acelasi numar de pixeli
  - interpretare:
    - inalta frecventa = detalii fine
    - frecventa joase = forma in general



Proprietati

transformare	*frontiera*	*descriptori Fourier*
identitate	$s(k)$	$a(u)$
rotatie	$s_r(k)=s(k)e^{j\theta}$	$a_r(u)=a(u)e^{j\theta}$
translatie	$s_t(k)=s(k)+\Delta_{xy}$	$a_t(u)=a(u)+\Delta_{xy}\delta(u)$
scalare	$s_s(k)=\alpha s(u)$	$a_s(k)=\alpha a(u)$
punct de start	$s_p(k)=s(k-k_0)$	$a_p(u)=a(u)e^{-j2\pi k_0 u/K}$

$\Delta_{xy}=\Delta x+j\Delta y$